Mechatronic Actuators

Lecture 5a

Asynchronous motors

Pointers

- Basics

$$A\cos(\omega t+\varphi)=\mathrm{Re}\left[Ae^{j(\omega t+\varphi)}\right]=\mathrm{Re}\left[Ae^{j\varphi}Ae^{j(\omega t)}\right]$$

- Transformation

$$x(t)=A\cos(\omega t+\varphi)\quad\rightarrow\quad\mathbf{X}(j\omega)=Ae^{j\varphi}$$

Impedance

- Definition

$$Z = \frac{\mathbf{U}(j\omega)}{\mathbf{I}(j\omega)}$$

- Resistor impedance

$$i(t) = \frac{U(t)}{R} = \frac{A}{R}\cos(\omega t)$$

$$Z_R = \frac{\mathbf{U}(j\omega)}{\mathbf{I}(j\omega)} = \frac{Ae^{j0}}{\frac{A}{R}e^{j0}} = R$$

Coil impedance

- Current across the coil

$$i(t) = \frac{1}{L}\int U\,dt = \frac{A}{L\omega}\sin(\omega t) = \frac{A}{L\omega}\cos\left(\omega t - \frac{\pi}{2}\right)$$

- Pointer

$$\frac{A}{L\omega}e^{-j\frac{\pi}{2}}$$

- Coil impedance

$$Z_L = \frac{\mathbf{U}(j\omega)}{\mathbf{I}(j\omega)} = \frac{Ae^{j0}}{\frac{A}{L\omega}e^{-j\frac{\pi}{2}}} = \frac{A}{-j\frac{A}{L\omega}} = j\omega L$$

Capacitor impedance

- Current across the capacitor

$$i(t) = C\frac{dU}{dt} = -CA\omega\sin(\omega t) = -CA\omega\cos\left(\omega t - \frac{\pi}{2}\right)$$

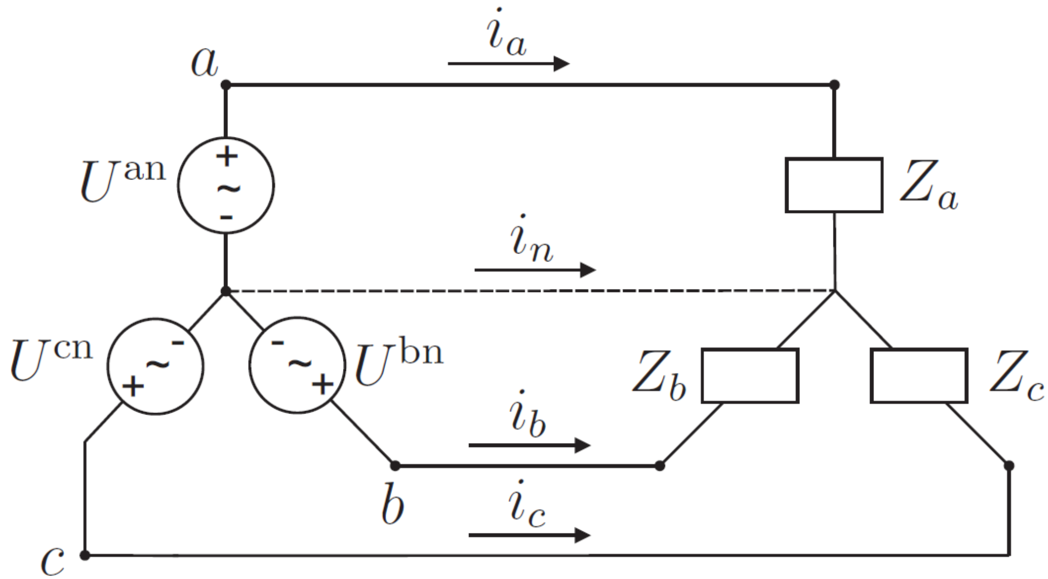
- Pointer

$$-CA\omega e^{j\frac{\pi}{2}}$$

- Capacitor impedance

$$Z_C = \frac{\mathbf{U}(j\omega)}{\mathbf{I}(j\omega)} = \frac{Ae^{j0}}{-CA\omega e^{-j\frac{\pi}{2}}} = \frac{1}{j\omega C}$$

3-phase voltage



$$\mathbf{U}_{\mathrm{ef}}^{\mathrm{an}}(j\omega) = U_{\mathrm{ef}}^{\mathrm{an}}\angle 0^\circ$$

$$\mathbf{U}_{\mathrm{ef}}^{\mathrm{bn}}(j\omega) = U_{\mathrm{ef}}^{\mathrm{bn}}\angle -120^\circ$$

$$\mathbf{U}_{\mathrm{ef}}^{\mathrm{cn}}(j\omega) = U_{\mathrm{ef}}^{\mathrm{cn}}\angle -240^\circ = U_{\mathrm{ef}}^{\mathrm{cn}}\angle 120^\circ$$

Characteristics of 3-phase voltage

- Power on load that is not just pure resistance

$$P(t) = U(t)i(t) = U_0 \cos(\omega t) i_0 \cos(\omega t - \varphi) = U_0 i_0 \frac{1}{2} \left[\cos\varphi + \cos(2\omega t - \varphi)\right]$$

- Average power

$$\bar{P} = \frac{U_0 i_0}{2} \cos\varphi = U_{\text{ef}} i_{\text{ef}} \cos\varphi = \frac{U_{\text{ef}}^2}{|Z|} \cos\varphi = i_{\text{ef}}^2 |Z| \cos\varphi$$

- Current power on three phases (resistors only)

$$P(t) = P_a(t) + P_b(t) + P_c(t) = \frac{U_{\text{ef}}^2}{R} \left[1 + \cos(2\omega t)\right] + \frac{U_{\text{ef}}^2}{R} \left[1 + \cos(2\omega t - 120^\circ)\right] + \frac{U_{\text{ef}}^2}{R} \left[1 + \cos(2\omega t + 120^\circ)\right] = \frac{3U_{\text{ef}}^2}{R}$$