Mechatronic Actuators

Lecture 3a

Basics of Electrical Engineering

Electric machines

Basics of electrical engineering

$$\oint \mathrm{D} \cdot \mathrm{dS} = \int \rho_e \, \mathrm{d}V = e$$

$$\nabla \cdot \mathrm{D} = \rho_e$$

$$\oint \mathrm{B} \cdot \mathrm{dS} = 0$$

$$\nabla \cdot \mathrm{B} = 0$$

$$\oint \mathrm{E} \cdot \mathrm{ds} = -\int \frac{\partial \mathrm{B}}{\partial t} \mathrm{dS} = -\frac{\mathrm{d}\Phi}{\mathrm{d}t}$$

$$\nabla \times \mathrm{E} + \frac{\partial \mathrm{B}}{\partial t} = 0$$

$$\oint \mathrm{H} \cdot \mathrm{ds} = \int \left(j_e + \frac{\partial \mathrm{D}}{\partial t} \right) \cdot dS = I + \frac{d\Phi_e}{dt}$$

$$\nabla \times \mathrm{H} - \frac{\partial \mathrm{D}}{\partial t} = j_e$$

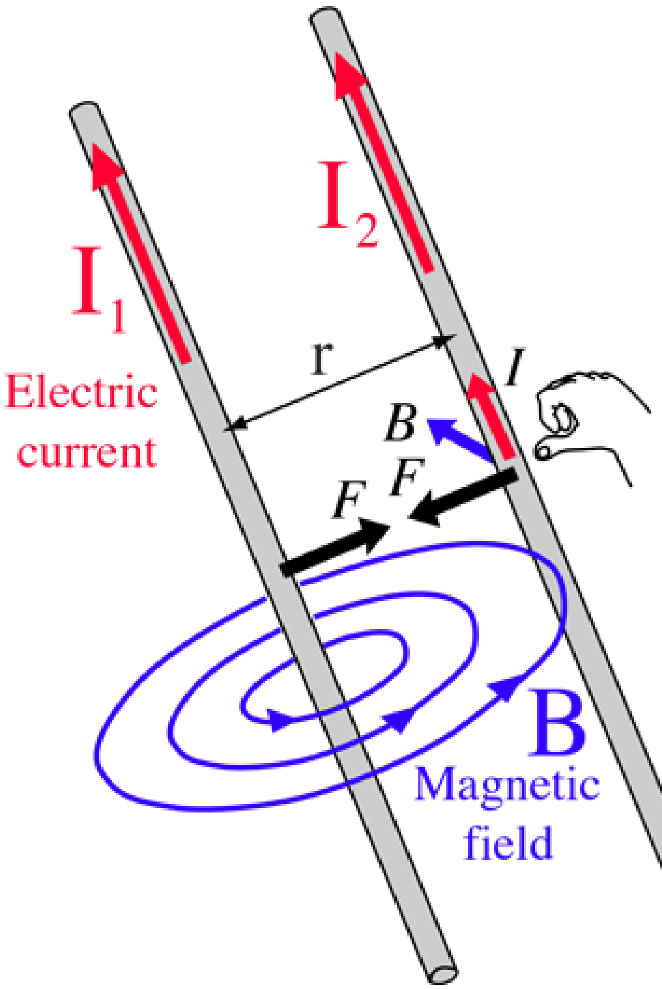
$\mathrm{D} = \varepsilon \varepsilon_0 \mathrm{E}$ $\qquad$ $\varepsilon_0 = 8.85\ 10^{-12} \frac{C^2}{Nm^2}$

$\mathrm{B} = \mu \mu_0 \mathrm{H}$ $\qquad$ $\mu_0 = 4\pi\ 10^{-7} \frac{H}{m}$

$\vec{\mathrm{F}} = -\frac{1}{4\pi\varepsilon_0} \frac{e_1 e_2}{\vec{\mathrm{r}}^2} \hat{r}$ $\qquad$ $\vec{\mathrm{F}} = e\vec{\mathrm{E}}$ $\qquad$ $\vec{\mathrm{E}} = -\frac{1}{4\pi\varepsilon_0} \frac{e}{\vec{\mathrm{r}}^2} \hat{r}$

$\vec{\mathrm{F}} = \frac{\mu_0}{4\pi} \frac{e_1 e_2}{\vec{\mathrm{r}}^2} \vec{\mathrm{v}} \times (\vec{\mathrm{v}}' \times \hat{r})$ $\qquad$ $\vec{\mathrm{F}} = q\vec{\mathrm{v}} \times \vec{\mathrm{B}}$ $\qquad$ $\vec{\mathrm{B}} = \frac{\mu_0}{4\pi} \frac{e_2}{\vec{\mathrm{r}}^2} (\vec{\mathrm{v}}' \times \hat{r})$

Magnetic field around a wire



Magnetic field at wire 2 from current in wire 1:

$$B = \frac{\mu_0 I_1}{2\pi r}$$

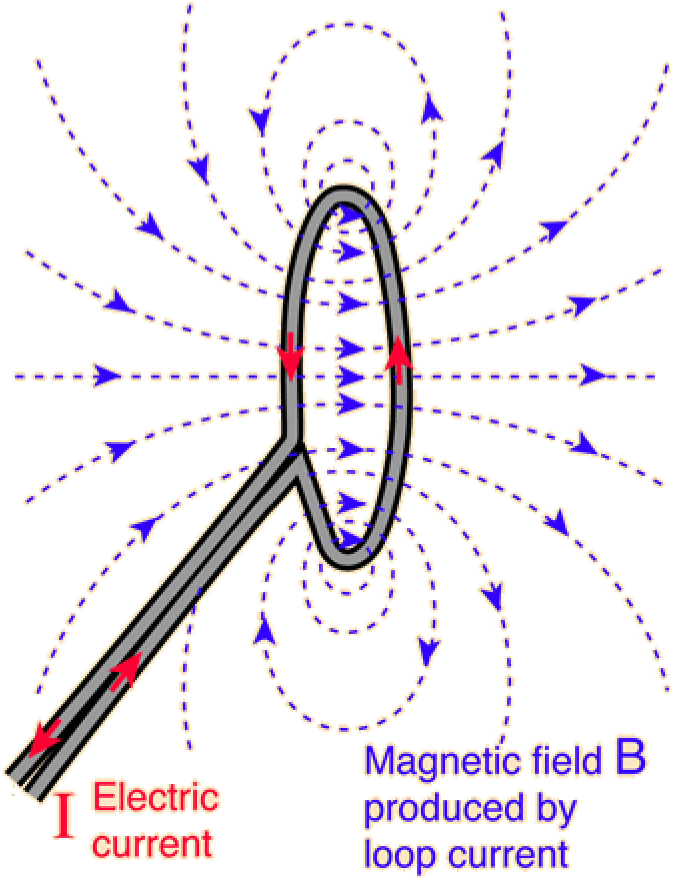
Force on a length ΔL of wire 2:

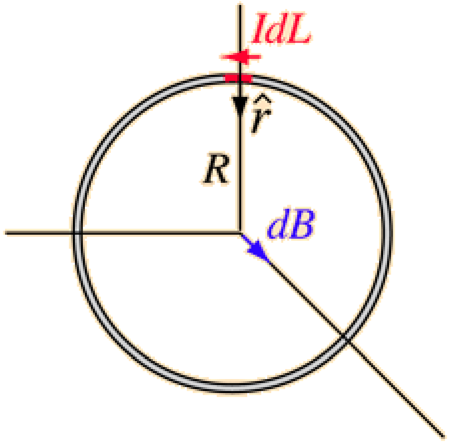
$$F = I_2 \Delta L B$$

Force per unit length in terms of the currents:

$$\frac{F}{\Delta L} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

Magnetic field of one loop of wire



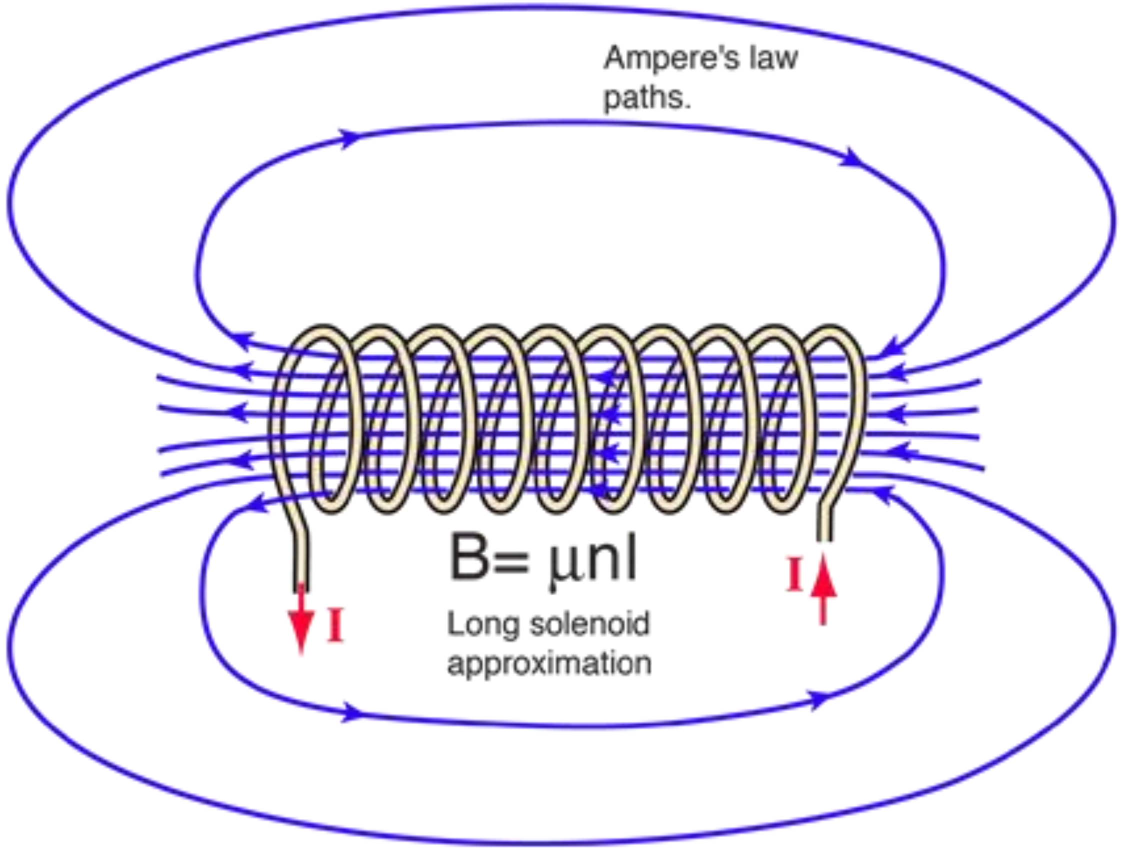


In this special case the symmetry is such that the field contributions of all the current elements around the circumference add directly at the center. The line integral of the length is just the circumference of the circle.

$$dB = \frac{\mu_0 I d\vec{L} \times \vec{\hat{r}}}{4\pi R^2} = \frac{\mu_0 I dL \sin\theta}{4\pi R^2}$$

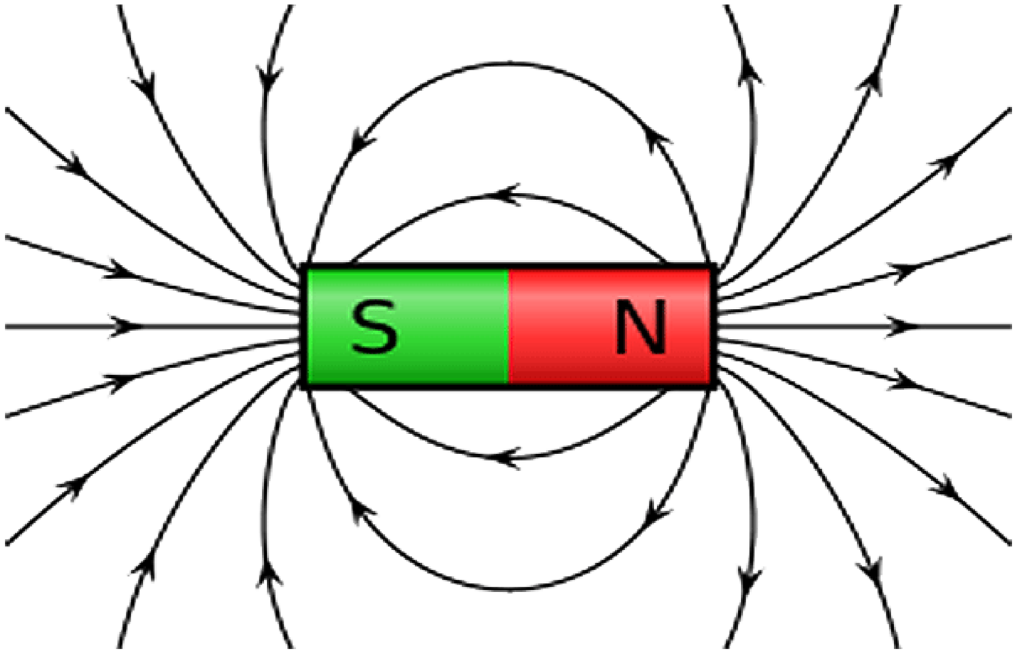
$$B = \frac{\mu_0 I}{4\pi R^2}\oint dL = \frac{\mu_0 I}{4\pi R^2} 2\pi R = \frac{\mu_0 I}{2R}$$

Magnetic field of a coil

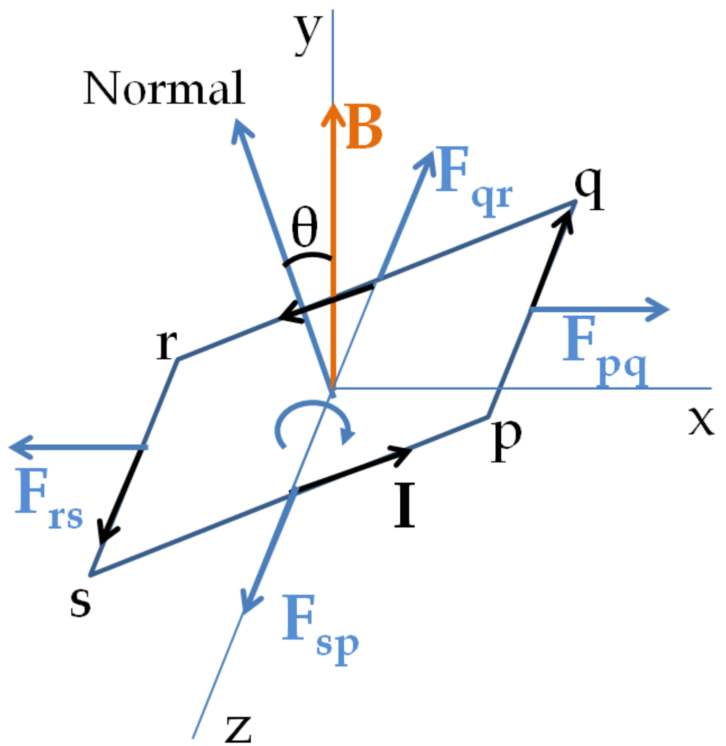


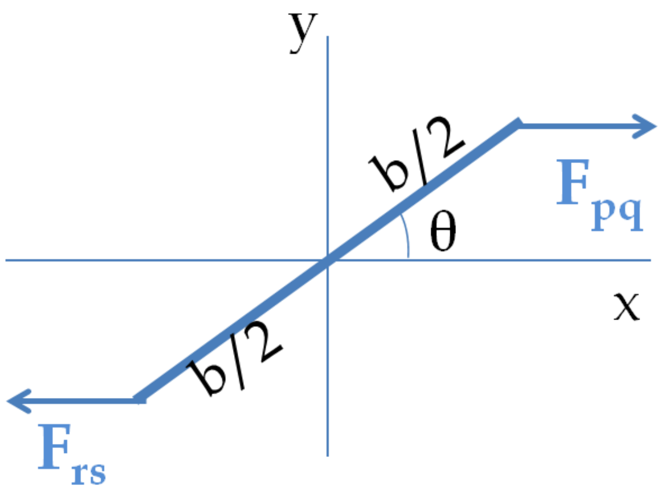
The magnetic field is concentrated into a nearly uniform field in the center of a long solenoid. The field outside is weaker and the lines representing the magnetic field are further apart.

Permanent magnet



Torque on a current loop

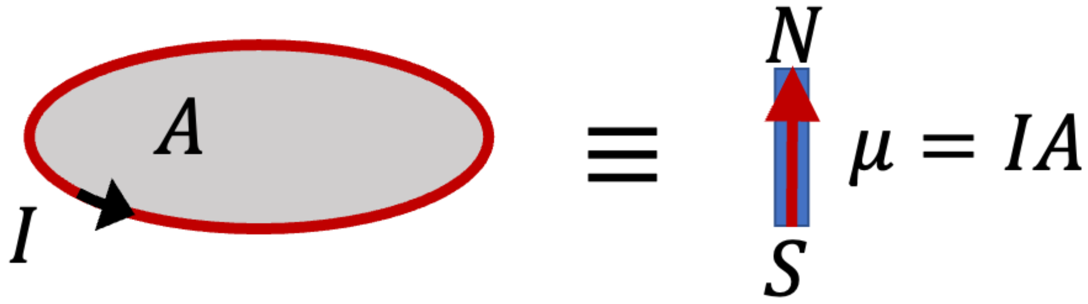




Magnetic dipole

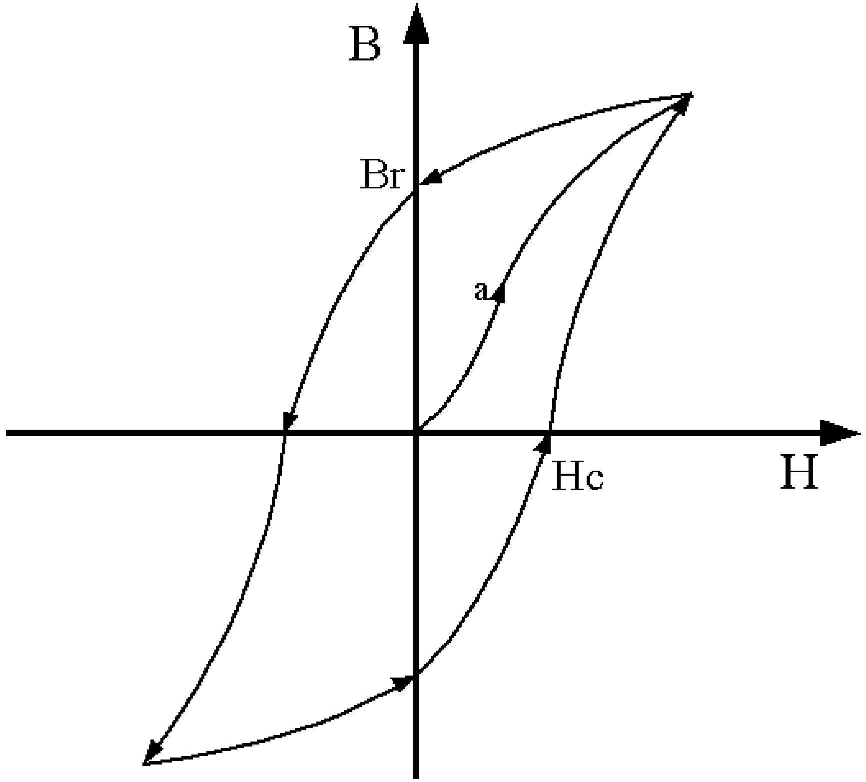
- $\tau = \mu \times B$

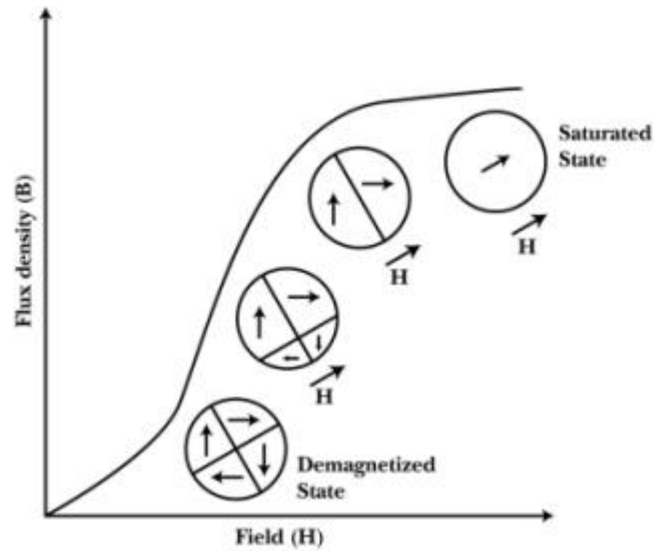
$$\mu_{\text{dip}} = IA$$



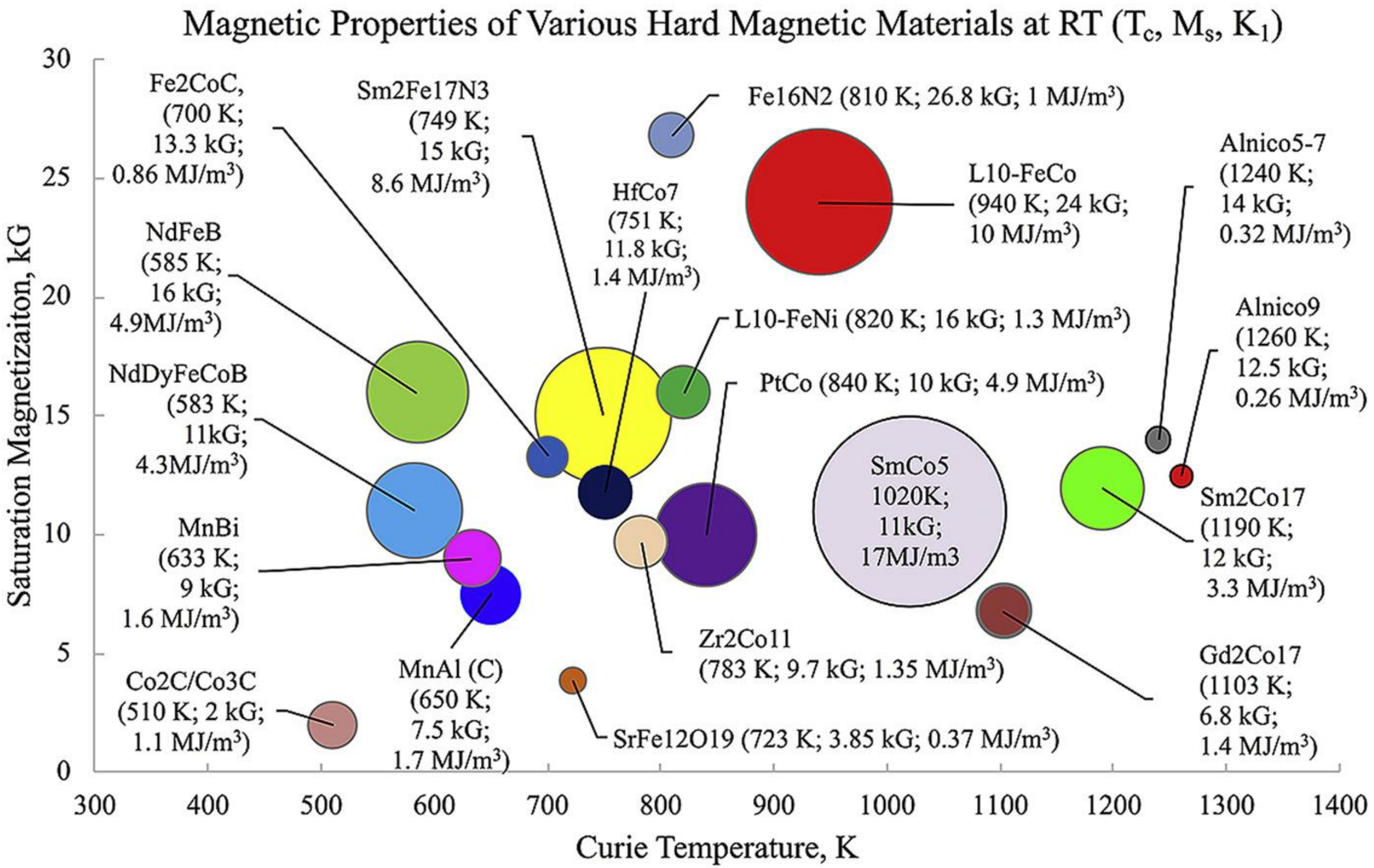
Hysteresis

The ferromagnetic material is selected according to th application requirements. For transformers, electromagnets, etc. we want a hysteresis loop as narrow as possible, B_r as small as possible and a characteristic that should resemble a straight line as much as possible. Such substances are called **soft magnetic substances**. When making a permanent magnet, we want a material with the highest possible B_r and the widest possible hysteresis loop, so that the material is as resistant as possible to the effects of the external magnetic field. Such substances are called **hard magnetic substances**. When manufacturing magnetic memory (tapes, disks, in the past also ferrite memory), we want B_r to be as large and as pronounced as possible, and at the same time, the hysteresis loop must be as narrow as possible, so that we do not use too much energy during state changes, which manifests itself in the heating of matter.





Magnetic properties of materials



Maximum current density

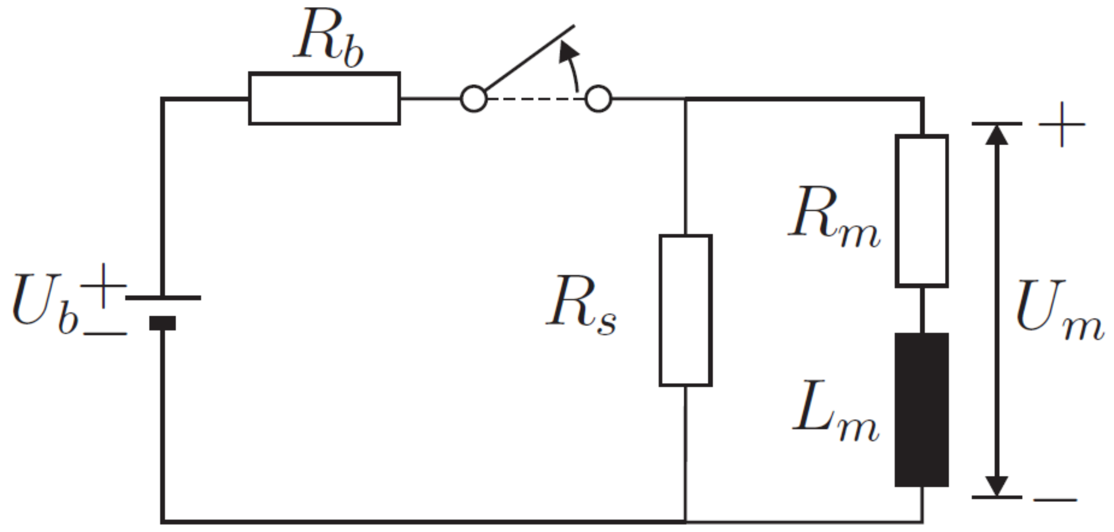
- For copper: $\rho = 95\,\frac{A}{mm^2}$

Approximate values for a motor

- Force acting on a conductor : $F = ilB_{max} = \bar{\imath}_{max} AB_{max} = \bar{\imath}_{max} 2\pi rlB_{max}$
- Torque: $M = Fr = \bar{\imath}_{max} 2\pi rlB_{max} r = k\bar{\imath}_{max} B_{max} V$
- Power: $P = M\omega = k\bar{\imath}_{max} B_{max} V\omega$

Problem

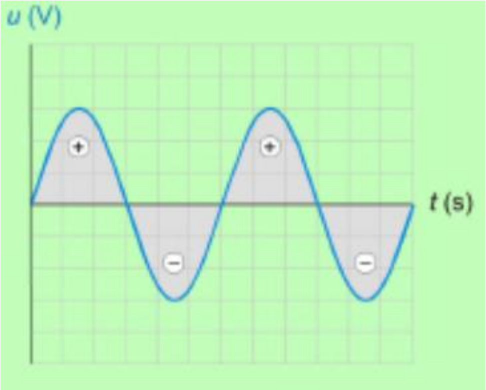
- The picture shows a simplified model of an electric motor. The switch is closed until time t = 0, and then it is opened. Determine the voltage on the electric motor U_m as a function of time if R_b = 2, R_s = 20, R_m = 1, L_m = 5H. The supply voltage is equal to U_b = 24V .



Typical voltage forms

- DC

- AC



Power grid voltage properties by country