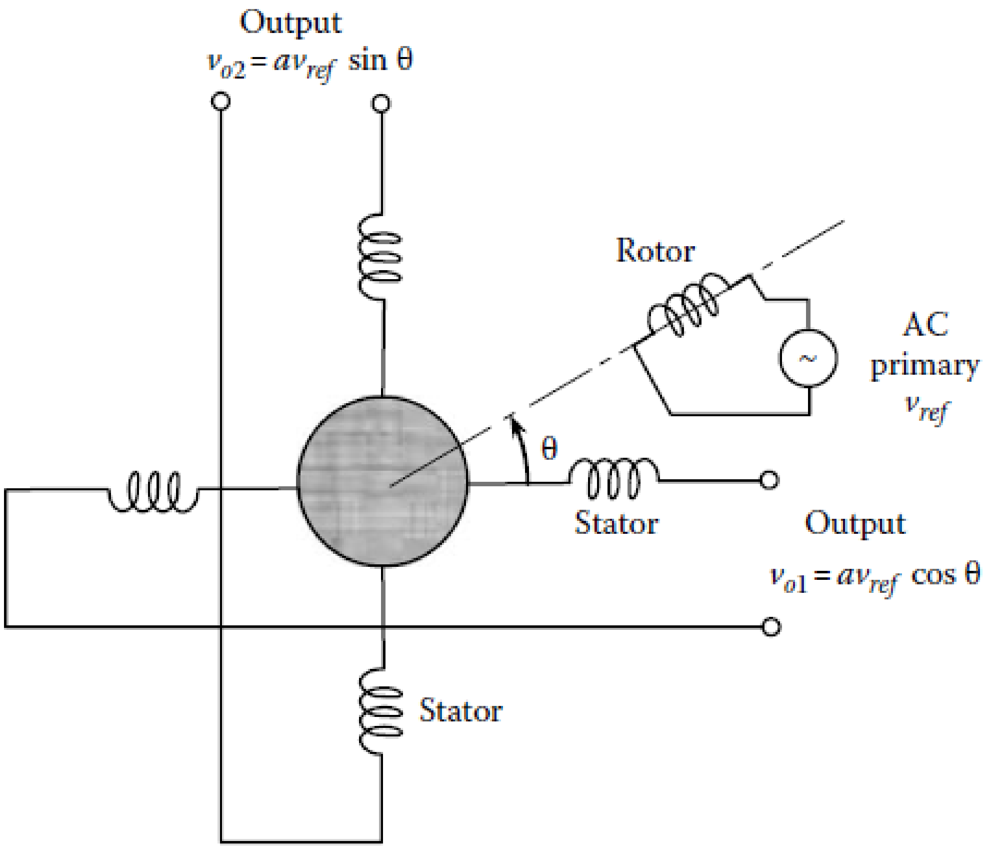
TASK 1

We use a **resolver** to measure the position, speed and direction of rotation of motors in demanding conditions (high rotation speeds, vibration, high temperatures, electromagnetic interference, etc.).

How does this device work and what are its output signals? How do we determine the key parameters of the motor rotation from the output signals?

Calculate the values of the output signals of the resolver for any angle Θ, if the sinusoidal voltage on the primary winding has an RMS value of $28\ V$ and a frequency of $400\ Hz$.

Schematic:



Data:

$U_{REF\ RMS}(t) = 28\ V$

$f = 400\ Hz$

$u_{OUT1}, u_{OUT2} = ?$

Solution:

A resolver is a type of sensor used in motion control applications for precise measurement of the position, speed, and direction of rotation of a rotating shaft. It is a completely analogue device that exploits the laws of electromagnetism, and in its construction, it resembles a motor at first glance.

The resolver consists of a rotor with primary winding and two pairs of stator windings. The rotor, with its winding, is mounted on the motor shaft. The rotor winding, which is the primary winding of the resolver, is excited with an alternating voltage$u_{REF} = U_0 \sin \omega t$

The rotor winding is excited with alternating voltage for several reasons:

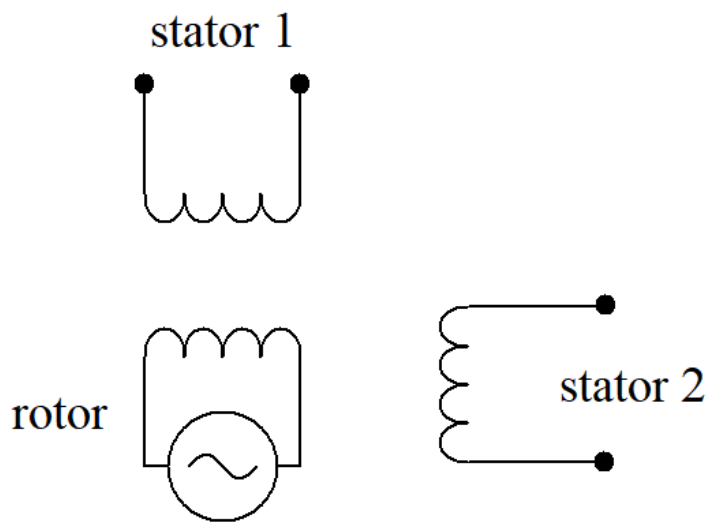
- phase sensitivity: Alternating voltage makes it easier to measure phase shifts between signals on the rotor and the stator (to determine the direction of rotation).
- signal processing: Alternating signals can be directly fed into analogue or digital circuits to determine the rotation angle; alternating signals simplify demodulation and filtering.
- inductive coupling of windings: If the rotor winding were excited with direct current, a magnetic field would form and eventually stabilize—the amplitude would be constant, losing phase information. This would prevent determining the motor's position and speed.
- noise and interference reduction: Alternating systems are easier to design to be resistant to noise and interference. For example, signals can be modulated at a specific frequency where the system is less sensitive to noise.

The stator is the stationary component surrounding the rotor and consists of two pairs of windings, offset by an angle of 90°. When voltage is applied to the rotor winding, current flows through it, creating a magnetic field in the winding. Since the rotor winding is mounted on the motor shaft, it rotates with the motor. As the winding rotates, so does its magnetic field, resulting in a rotating magnetic field. Near the motor and rotor, two pairs of stator windings are positioned at right angles to each other. From electrical engineering, we recall that any change in the magnetic field (magnetic flux) within a winding causes electromagnetic induction. Because the stator windings lie within the magnetic field region of the rotor and the rotor rotates, the magnetic flux within the windings changes. The equation for electromagnetic induction is

$$u_i = -N \frac{d\Phi}{dt}$$

where U_i is the voltage induced at the winding terminals, N is the number of turns in the winding, and Φ is the magnetic flux within the winding. Any change in magnetic flux causes induction: the magnetic flux density may change, the cross-sectional area of the winding may change, or the relative position of the winding with respect to the source of the magnetic field may change. In our case, the latter is the reason for the change—since the rotor winding rotates relative to the stator winding, the magnetic flux in both pairs of stator windings changes. The result is induced voltages in the stator windings. The goal is to determine the equations for these induced voltages as a function of the rotor angle Θ.

For easier further analysis, let us draw the resolver diagram slightly differently, sketching a "top view." The rotor winding lies in the plane of the sheet, with the stator windings positioned at a 90° angle.



The change in magnetic flux in the stator windings is greatest when the stator winding is parallel to the rotor winding. If the rotor winding is initially positioned horizontally and stator winding 1 is positioned as shown in the top view sketch (also horizontally), at this moment, the change in magnetic flux in winding 1 is at its maximum. When the rotor winding rotates by 90°, it is perpendicular to stator winding 1. At that time, the change in magnetic flux is zero. We see that the voltage in stator winding 1 varies with the cosine of the rotor angle Θ. Similarly, for stator winding 2, the induced voltage varies with the sine of the angle Θ. We can write:

$$u_1 = u_{REF} \cdot a \cdot \cos \Theta$$

$$u_2 = u_{REF} \cdot a \cdot \sin \Theta$$

where a is a factor that depends on the winding (number and cross-sectional area of the windings).

The magnetic flux is generally calculated using the surface integral of the magnetic flux density over a closed surface.

$$\Phi = \oint B\,dA$$

If the winding is in the form of a rectangle and the magnetic flux density does not change across the surface (a case we studied in electrical engineering), the calculation of magnetic flux simplifies to:

$$\Phi = \oint B\,dA = B \cdot A = B \cdot a \cdot b$$

Since the winding rotates, the exact form of the magnetic flux signal is:

$$\Phi = B \cdot a \cdot b \cos \omega t$$

If we substitute this equation into the equation for induced voltage, we get:

$$u_i = -N \frac{d\Phi}{dt} = N \cdot B \cdot a \cdot b \cdot \omega \cdot \sin \omega t$$

For the resolver considered in this task, the above equation changes in that the winding is not rectangular but rather a wire wound in a circular shape, altering the equation for the area of the winding surface. Another difference is that the magnetic flux density of the rotor's magnetic field changes over time because the rotor winding is excited with alternating voltage. We could write the above equation as:

$$u_i = N \cdot B(t) \cdot \pi \cdot r^2 \cdot \omega \cdot \sin \omega t$$

From the above equation, it is evident what affects the induced voltage, and we have already simplified the equation in the previous record, as we are mainly interested in the temporal dependence of the induced voltage on the excitation voltage on the primary winding and the rotor angle. Let us write:

$$u_1 = U_0 \cdot a \cdot \cos \Theta(t) \sin \omega t$$

$$u_2 = U_0 \cdot a \cdot \sin \Theta\,(t) \sin \omega t$$

$\sin \omega t$ is the carrier signal on the rotor winding and is generated by an alternating source. The frequency of this signal is usually ten times higher than the frequency (speed) of the motor's rotation. The above signals could also be described as amplitude-modulated signals. The amplitude of the carrier signal is modulated by the cosine and sine of the rotation angle.

How do we obtain the rotation angle, direction of rotation, and speed from the two signals on the stator windings? A component of the resolver is an electronic circuit that calculates the required quantities from the induced voltage signals. The rotation angle could be calculated using the inverse trigonometric function (arc cosine or arc sine) from one of the signals on the stator winding. A more suitable approach is to divide the induced signal on stator winding 2 by the induced signal on stator winding 1. All terms except those dependent on the rotation angle cancel out, yielding a ratio equal to the tangent of angle Θ. Using the arctangent operation, we get the actual rotation angle. This method is interesting because dividing the two signals reduces potential noise in the induced voltages and allows for temperature compensation.

When determining the angle, we must distinguish between four possible situations:

- Angle between 0 and 90°:
    - The cosine value is positive, and the sine value is negative
- In this case, one of the stator signals is sufficient to determine the angle, as there is no ambiguity
- Angle between 90 and 180°
    - Cosine is negative, and sine is positive
- Angle between 180 and 270°
    - Cosine is negative, and sine is negative
- if we used the signal on the first stator winding for angles between 90 and 270 degrees, we wouldn't be able to determine the angle accurately, as cosine is negative in both quadrants
- Angle between 270 and 360°
    - Cosine is positive, and sine is negative
- If we used the signal on the second stator winding for angles between 180 and 360 degrees, we wouldn't be able to determine the angle accurately, as sine is negative in both quadrants

For differential resolvers, the angle and direction of rotation can be determined through demodulation. First, we multiply both stator winding signals by the signal on the rotor winding:

$$\begin{aligned} u_{m1} = u_1 \cdot u_{REF} = U_0 \cdot a \cdot \cos\Theta(t) \sin\omega t \cdot U_0 \ \sin\omega t &= U_0^2 \cdot a \cdot \cos\Theta(t) \sin\omega t^2 \\ &= U_0^2 \cdot a \cdot \cos\Theta(t)\,(1 - \sin 2\omega t) \end{aligned}$$

$$u_{m2} = u_2 \cdot u_{REF} = U_0 \cdot a \cdot \sin \Theta(t) \sin \omega t \cdot U_0 \; \sin \omega t = U_0^2 \cdot a \cdot \sin \Theta(t) \sin \omega t^2$$
$$= U_0^2 \cdot a \cdot \sin \Theta(t) \, (1 - \sin 2\omega t)$$

Both results consist of two components: the first shows the dependence on the rotation angle Θ, and the second includes the carrier signal $\sin \omega t$. Since the carrier signal is not important for determining the rotation angle, we can remove it using a low-pass filter, which cuts off the carrier signal frequency. The demodulated signals are:

$$u_{d1} = U_0^2 \cdot a \cdot \cos \Theta(t)$$

$$u_{d2} = U_0^2 \cdot a \cdot \sin \Theta(t)$$

From these two signals, we can determine the motor's rotation angle and direction of rotation.

Advantages of the resolver:

- high accuracy and resolution (approximately 0.01°)
- low output impedance
- small size (approximately 10 mm in diameter)
- reliability and durability (electromagnetic principle and contactless sensing)
- resistance to vibrations, shock, high temperatures, electromagnetic interference
- suitability for high rotation speeds

Limitations:

- higher cost due to more complex construction (shaft mounting, windings)
- bandwidth is limited by the carrier signal frequency
- not suitable for linear motor position control

Typical applications:

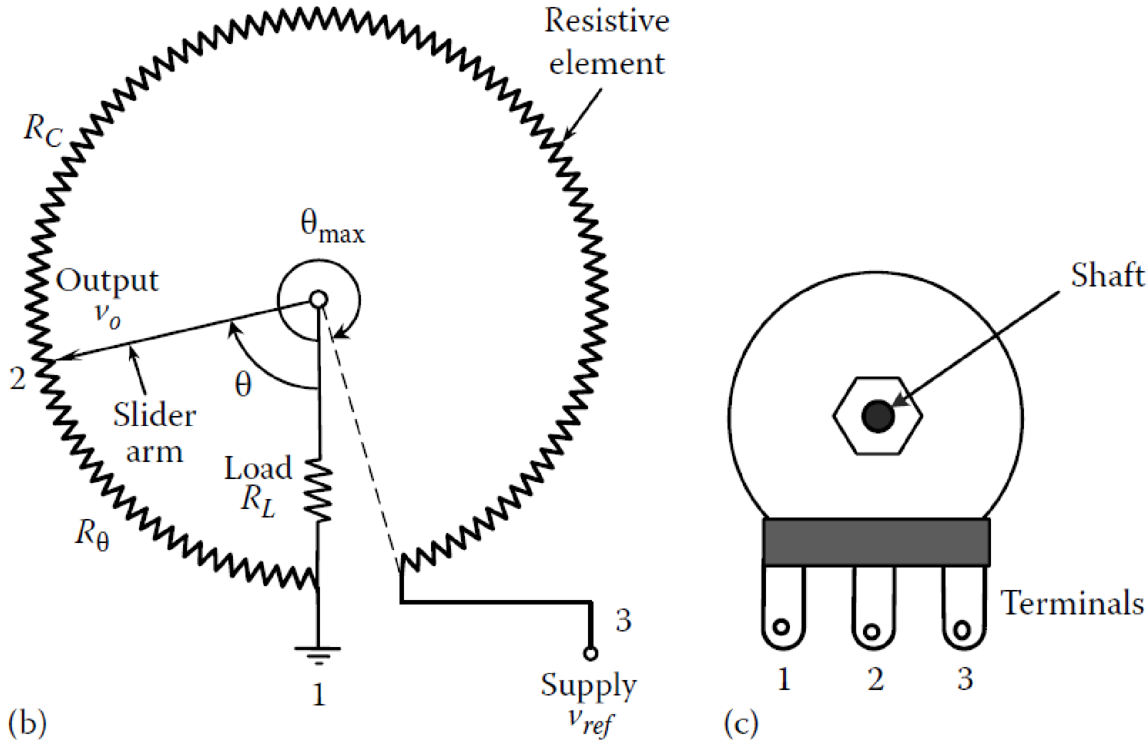
- industrial automation
- robotics
- electric vehicles
- space technologies

TASK 2

How does a **rotary potentiometer** work to determine the motor position? Explain the load nonlinearity problem of a rotary potentiometer and its solution.

If a rotary potentiometer is made as a coil of wire with a thickness of $0.1\ mm$ and the operating range of the potentiometer (coil) is $10\ cm$, what is the resolution of such a potentiometer? What is the voltage on the applied part of the winding at the smallest turn that we can measure with such a potentiometer, if the reference voltage is $10\ V$? Let us assume that the potentiometer allows measurements of rotations between 0 and 355°.

Schematic:



Data:

$d = 0.1\ mm$

$l = 10\ cm$

$U_{REF} = 10\ V$

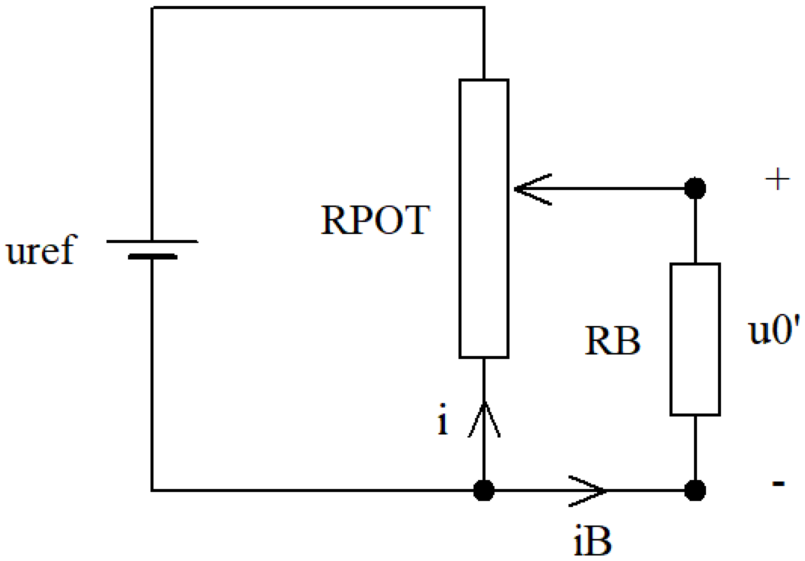
$r, U_\Theta = ?$

Solution:

A potentiometer is a device that essentially functions as a variable resistor or a voltage divider. The typical design includes three terminals and a wire winding. Two terminals are fixed at both ends of the winding, while the third terminal is connected to a slider that can move along the winding. This allows a portion of the winding to be selected, adjusting the desired resistance. For example, this enables the adjustment of the voltage on the connected load.

A rotary potentiometer is a circular version of the potentiometer. The wire winding is arranged in a circle around the motor shaft. The slider is attached to the shaft and moves with it. The movement of the slider follows the rotation of the motor shaft, reflecting the rotation angle of the motor. The potentiometer has its own DC power supply, which serves as a reference voltage. The output signal of the potentiometer is the voltage between the sliding terminal and the reference voltage terminal. This signal is a proportional part of the reference voltage depending on the resistance of the part of the potentiometer winding relative to the slider position.

Let's simplify the diagram of the rotary potentiometer:



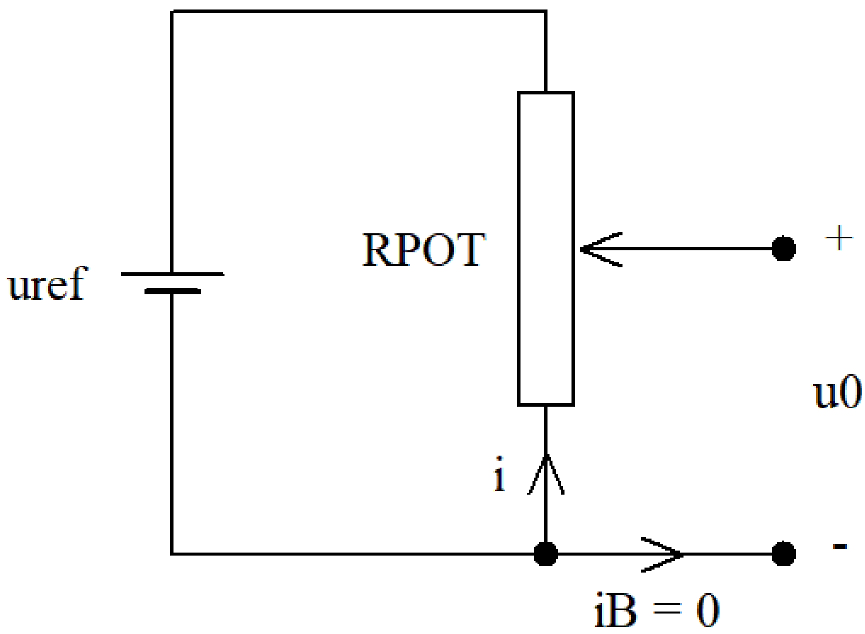
Let's denote the resistance of the entire winding of the potentiometer as R_{POT}. This resistance can range from 10 Ω up to 1 MΩ. The resistance of the winding between the ground terminal and the slider terminal is denoted as R_{Θ}. The rotation angle of the motor Θ j is the angle between the slider and the ground.

Due to its construction, the rotary potentiometer has an important limitation. The winding cannot cover the full 360° angle but always slightly less; in our case, the maximum angle $\Theta_{MAX} = 355°$. Therefore, the resistance of the entire winding R_{POT} corresponds to the angle Θ_{MAX}. One possible solution is a spiral design of the potentiometer winding, where the basic winding is placed in a spiral shape in multiple circles.

When calculating the output voltage of the potentiometer, we must also consider the load resistance R_B, which represents the resistance of the circuit following the potentiometer circuit. The output signal of the potentiometer needs to be processed so it can be used for control purposes. Let's examine the problem with two examples. If no load is connected to the potentiometer, the output voltage is:

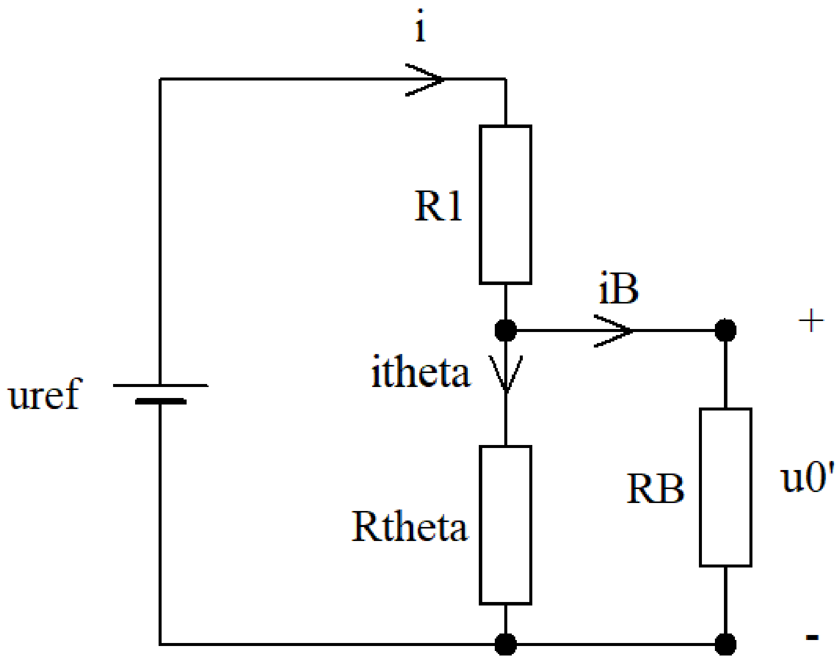
$u_0 = k\Theta$

In this case, the load resistance is infinitely large, the output terminals are open, and no current flows from the potentiometer.



In a real scenario, the load resistance will have a finite value, and there will be some current flowing through the potentiometer's output. Since the load resistance is connected in parallel with the resistance R_Θ, the total resistance at the output will be lower, thus reducing the voltage. The linear equation for u_0 ta no longer applies, and using it would yield incorrect results. The effect of load resistance causes a so-called loading nonlinearity, which we will determine by deriving equations.

To determine the output voltage, let us slightly modify the potentiometer circuit diagram:



The resistance of the winding corresponding to the motor's rotation angle can be expressed as:

$$R_{\Theta} = \frac{\Theta}{\Theta_{MAX}} R_{POT}$$

For the node at the potentiometer's sliding contact, we write Kirchhoff's current law equation:

$$i = i_{\Theta} + i_B$$

Considering the above diagram, we express the currents with voltages:

$$\frac{u_{REF} - u_0}{R_{POT} - R_{\Theta}} = \frac{u_0}{R_{\Theta}} + \frac{u_0}{R_B}$$

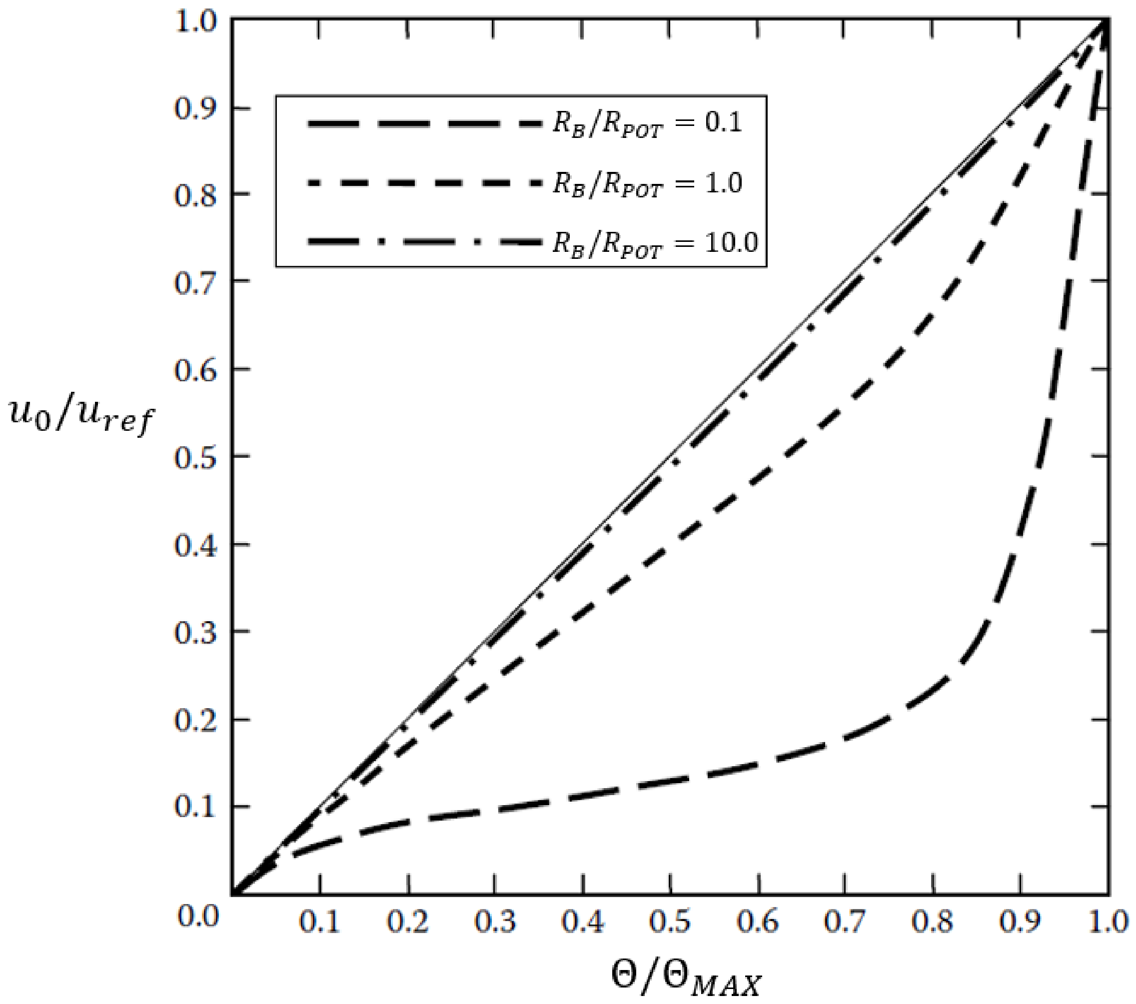
By rearranging the equation, we get the voltage divider equation or the ratio of the potentiometer's output voltage to the reference voltage:

$$\frac{u_0}{u_{REF}} = \frac{\frac{\Theta}{\Theta_{MAX}} \frac{R_B}{R_{POT}}}{\frac{R_B}{R_{POT}} + \frac{\Theta}{\Theta_{MAX}} - (\frac{\Theta}{\Theta_{MAX}})^2}$$

The error due to loading nonlinearity is calculated as:

$$e = \frac{\frac{u_0}{u_{REF}} - \frac{\Theta}{\Theta_{MAX}}}{\frac{\Theta}{\Theta_{MAX}}} 100\%$$

The nonlinearity error can be clearly illustrated in a graph:



The graph shows that the nonlinearity error is particularly pronounced at small ratios of $\frac{R_B}{R_{POT}}$, to i.e., when the load resistance is small compared to the total resistance of the potentiometer. In such cases, the error can exceed 70%. Therefore, to reduce the nonlinearity problem, we can ensure higher resistance on the potentiometer's output side, typically $\frac{R_B}{R_{POT}} > 0$. This can be achieved, for example, by adding a resistor or using a voltage follower with high input impedance, which virtually eliminates the nonlinearity.

Next, let us determine the resolution of the given potentiometer. The resolution is determined by the construction of the potentiometer – the slider jumps from one turn of wire to the next, defining the minimum angle the potentiometer can detect. Therefore, the potentiometer has a finite resolution in every case, conditioned by the number of turns in the winding. The resolution can be determined as:

$$r = \frac{100}{N}\%$$

where the resolution r represents the percentage of the entire range of the output value, and N is the number of turns in the potentiometer's winding. For example, if we had a potentiometer with 100 turns and it could measure the rotation angle over the full circle, its resolution would be:

$$r = \frac{360°}{100} = 3.6°$$

In our case, the range of possible rotation angle measurements is limited to 355°. The number of winding turns will be determined by the total length of the winding, which is 10 cm, and the wire thickness in the winding, which is 0.1 mm. Assuming the wire is wound so tightly that the turns practically touch each other, we can approximate the number of turns as:

$$N \doteq \frac{10\ cm}{0.1\ \ mm} = 1000$$

Therefore, the resolution of our potentiometer is:

$$r = \frac{355°}{1000} = 0.355°$$

The voltage on the used part of the winding at the smallest rotation, which can be measured with such a potentiometer, is:

$$U_{MIN} = \frac{0{,}355\ °}{355°} = \frac{10}{1000} 10\ V = 10\ mV$$

In practice, potentiometers with a resolution of less than 0.1% (1000 turns in the winding) can be found. For even better resolution, optimized designs can be used, such as those with conductive plastic on the sliding contacts.

Advantages of potentiometers compared to other methods of measuring motor rotation:

- lower cost
- simpler manufacturing
- high resolution
- capability of linear measurements
- low sensitivity to electrical noise

Potentiometers also have some important drawbacks or limitations:

- lower durability, especially in demanding environments (moisture, temperature, vibrations)
- limited range of motion (mostly not designed for full rotation)
- mechanical contact on the slider → wear and tear
- not suitable for high speeds → the slider cannot follow rapid motor rotations

TASK 3

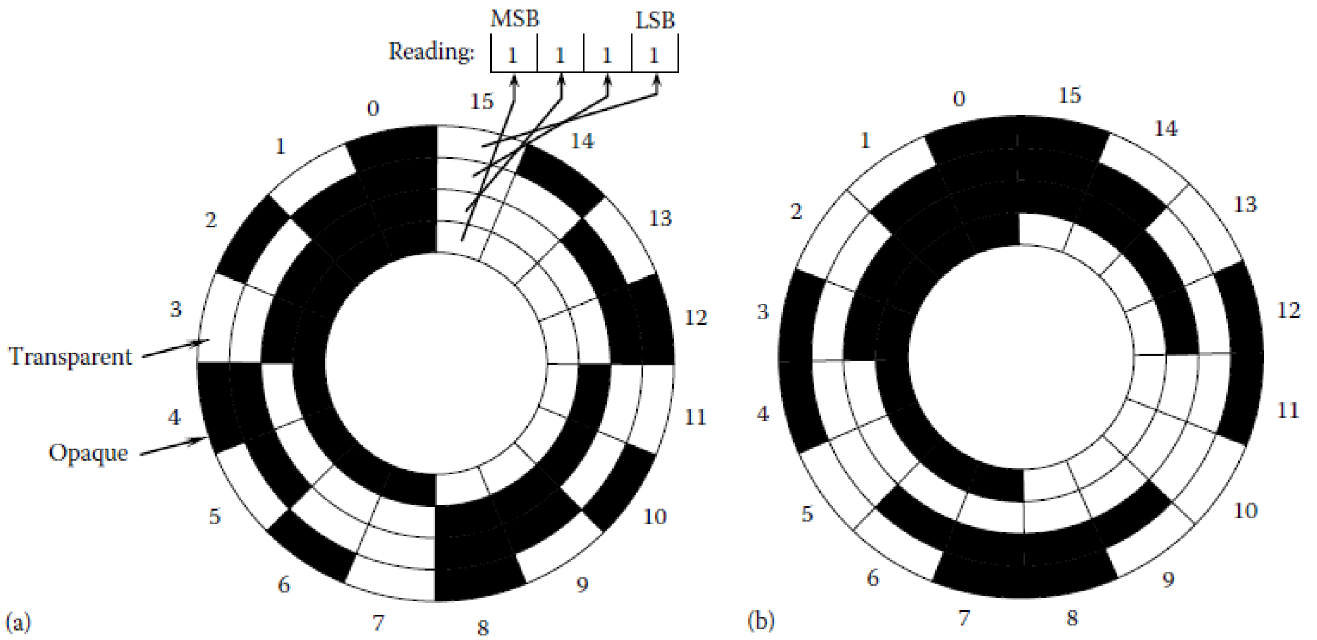
Demonstrate the different ways of encoding the motor position for a 4-bit **absolute encoder**:

a) Binary encoder

b) Gray encoder (binary mapped Gray encoder)

What are the differences between the two encoders? What are the main advantages of a Gray encoder that are useful in determining the absolute position of a motor?

Schematic:



Solution:

An absolute encoder is a device that enables the precise determination of the motor's position. The output data of the encoder directly represents the motor's position (rotation angle of the shaft).

An absolute encoder generally consists of:

- an encoding disk
- a control signal source
- a control signal sensor
- electronics

Possible implementations of the control signal source and sensor are:

- optical encoder: The control signal is light, detected by a photodiode or phototransistor.
- magnetic encoder: Magnetic sensors detect magnetic reluctance.
- PIR encoder: Proximity capacitive sensors detect changes in capacitance.
- sliding contact encoder: Operates on the principle of electrical conductivity.

The most common type is the optical encoder.

The encoding disk is circular, with multiple levels of sections that either allow or block light. The control signal source (usually light) generates the signal and sensors appropriately positioned on each circle detect the received light. This creates a binary pulse train: at any given moment, a section on the encoding disk either allows light to pass through or does not. Therefore, the pulses can only have two values, representing the binary values 0 and 1. The arrangement of the sections depends on the type of encoding. Each circle represents 1 bit of information, which together form the binary representation of the motor's position (rotation angle).

An absolute encoder mounted on the motor shaft, in addition to the encoding disk, also has as many sensors as there are circles (rings). If a 4-bit encoder is used, the encoding disk has four circles, and there are four sensors that detect light on the sections of the circles. Unlike an incremental encoder, an absolute encoder does not count pulses but directly determines the value corresponding to the motor's position from the output signals of the photo sensors.

A 4-bit encoder produces one of $2^4 = 16$ values at the output. This directly determines the encoder's resolution:

$$r = \frac{360°}{2^4} = 22.5°$$

In practice, 10-bit or 12-bit encoders are used, which provide much better measurement resolution:

$$r = \frac{360°}{2^{12}} = 0.0879°$$

The classic principle of an absolute encoder is a binary encoder. Each possible rotation angle value is determined by its binary form, obtained by converting the decimal form of an integer to binary. For example, the value 13 is represented in binary as:

$$13 = 1 \cdot 8 + 1 \cdot 4 + 0 \cdot 2 + 1 \cdot 1 = 1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 = 1101$$

On the encoding disk of an absolute encoder, values are read so that the innermost ring represents the most significant bit (MSB), and the outermost ring represents the least significant bit (LSB). For example, if the sensor (photodiode) on the innermost ring detects an opaque section (bit value 0), on the next ring also an opaque section (value 0), and on the remaining two rings sections that allow light to pass through (bit value 1), this represents the value:

$$0011 = 0 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 = 3$$

Let's create a table with all possible combinations of a 4-bit binary encoder:

Decimal	Classic binary encoding			
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
10	1	0	1	0
11	1	0	1	1
12	1	1	0	0
13	1	1	0	1
14	1	1	1	0
15	1	1	1	1

Analysing the table and considering the electronics that read the encoding disk, we encounter a major problem. As the motor and the encoding disk rotate, the electronics read values in sequence on the encoding disk. When transitioning from decimal value 1 (0001) to value 2 (0010), two bits change in the binary representation. When transitioning from value 3 (0011) to value 4 (0100), three bits change. This presents a challenge for the measurement circuit, as these transitions are not instantaneous. It is possible that the photodiode on one ring of the encoding disk has already detected a transition, while the photodiode on another ring is still detecting the previous value. During transitions, the encoder's output values can be undefined and completely incorrect.

The solution lies in a different method of encoding decimal to binary values. Gray code is an example of encoding where only 1 bit changes with each detected rotation angle. We illustrate the operation with a table:

<table>
<tr><th>Decimal</th><th colspan="4">Gray code</th><th colspan="4">Encoder</th></tr>
<tr><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td rowspan="2">1 bit encoder</td><td rowspan="4">2 bit encoder</td><td rowspan="8">3 bit encoder</td><td rowspan="16">4 bit encoder</td></tr>
<tr><td>1</td><td>0</td><td>0</td><td>0</td><td>1</td></tr>
<tr><td>2</td><td>0</td><td>0</td><td>1</td><td>1</td><td rowspan="14"></td></tr>
<tr><td>3</td><td>0</td><td>0</td><td>1</td><td>0</td></tr>
<tr><td>4</td><td>0</td><td>1</td><td>1</td><td>0</td><td rowspan="12"></td></tr>
<tr><td>5</td><td>0</td><td>1</td><td>1</td><td>1</td></tr>
<tr><td>6</td><td>0</td><td>1</td><td>0</td><td>1</td></tr>
<tr><td>7</td><td>0</td><td>1</td><td>0</td><td>0</td></tr>
<tr><td>8</td><td>1</td><td>1</td><td>0</td><td>0</td><td rowspan="8"></td></tr>
<tr><td>9</td><td>1</td><td>1</td><td>0</td><td>1</td></tr>
<tr><td>10</td><td>1</td><td>1</td><td>1</td><td>1</td></tr>
<tr><td>11</td><td>1</td><td>1</td><td>1</td><td>0</td></tr>
<tr><td>12</td><td>1</td><td>0</td><td>1</td><td>0</td></tr>
<tr><td>13</td><td>1</td><td>0</td><td>1</td><td>1</td></tr>
<tr><td>14</td><td>1</td><td>0</td><td>0</td><td>1</td></tr>
<tr><td>15</td><td>1</td><td>0</td><td>0</td><td>0</td></tr>
</table>

The table shows the possibilities for using Gray code encoders from 1-bit to 4-bit configurations. In each transition, only one bit changes, even when reaching the maximum value and returning to the lowest value.

Other possible solutions to the problem of incorrectly detected states in regular binary encoding include:

- implementing a check for the value of two consecutive sensor detections – since the encoding table is known, a transition is possible only to one of two adjacent states. If the photo sensors detect another value, this reading can be discarded as incorrect.
- using a delay element, such as a Schmitt trigger – detection is delayed until the transition is complete on all circles of the encoder.

Advantages of the absolute encoder:

- precise position determination without a reference point

- ability to measure rotation angle for multiple independent axes without the need for synchronization
- immediate information of the motor's rotation angle after a power outage
- useful in demanding environments, such as in the presence of vibrations, contamination, etc.
- better resolution than incremental encoders

Disadvantages of the absolute encoder compared to incremental encoders include:

- higher cost
- more complex electronics
- suitable for lower speeds
- sensitivity to damage to the encoding disk and sensors

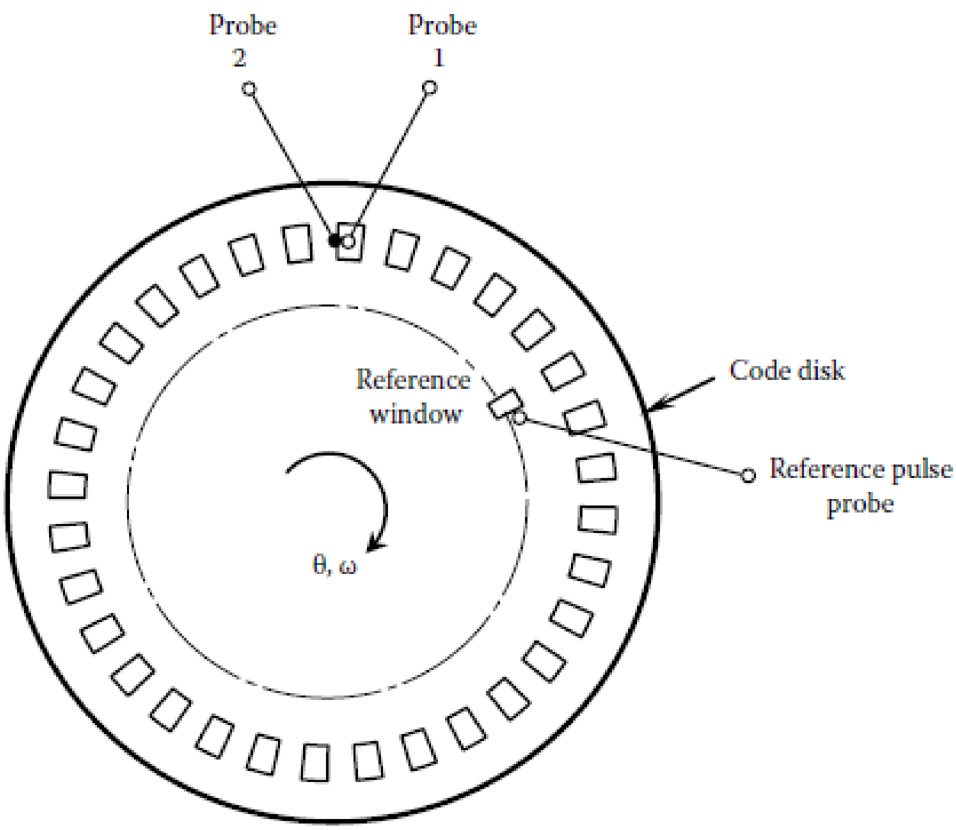
TASK 4

How does an **incremental encoder** work? How many signals does it need to use to accurately determine the position or speed of the motor and why?

Let us imagine an ideal 12-bit incremental encoder that measures the speed of the motor. The encoder has 500 windows per centimetre of encoder disc diameter. Determine the equation for the digital and physical resolution of the encoder and calculate the corresponding diameter of the encoder disc assuming the use of quadrature signals.

At time $T = 5$, we count 1000 pulses with the help of the encoder disc. What is the engine speed?

Schematic:



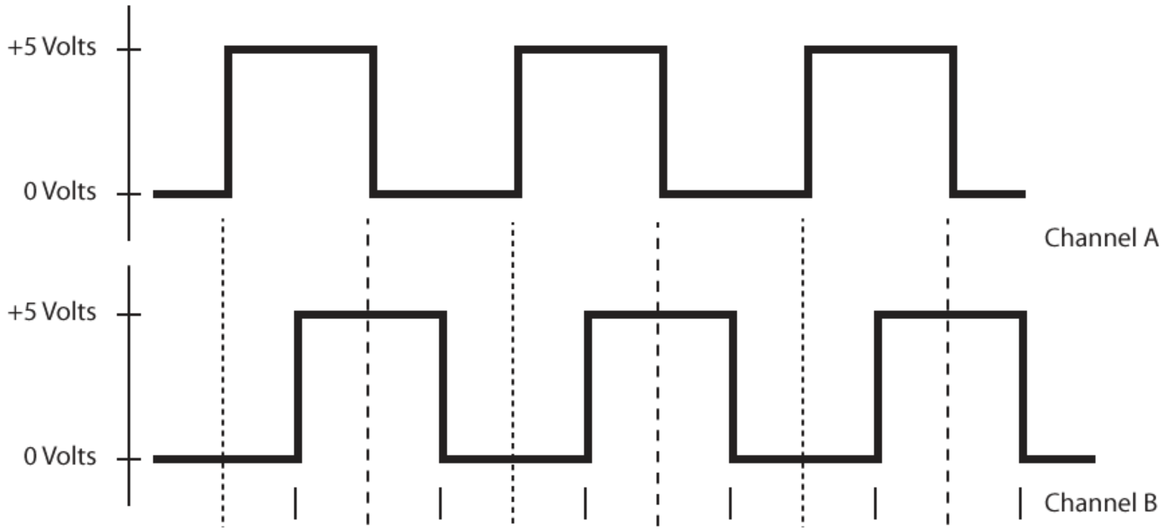
Data:

$r = 12\ bits$
$w = 500\ /\ cm$
$T = 5\ s$
$n = 1000$

$\Delta\Theta_d, \Delta\Theta_p, d = ?$
$\omega = ?$

Solution:
An incremental encoder is a device used to measure the position of a motor. It cannot determine the exact rotation angle of the motor, but only the change in angle. To function, an incremental encoder requires a reference point or a reset that sets it to a known initial position.

An incremental encoder needs three signals to determine the rotation angle and speed (e.g., three photo sensors). Two signals are obtained from two sensors that detect light through transparent windows on the main ring of the encoding disk (channel A and channel B).



These channels are offset by 90°. To determine the (change in) rotation angle of the motor, the pulses detected by the sensors during rotation are counted. The rotation direction is determined by observing the order in which the two sensors are triggered. If channel A is 90° phase (time) ahead of channel B and the sensor on channel B triggers before the sensor on channel A, the motor rotates clockwise. The third signal is a reference signal obtained from a sensor on an additional ring of the encoding disk. This ring has only one transparent window. Triggering the sensor that detects this window provides a reference point from which changes in angle are measured.

In this task, we use a 12-bit incremental encoder that uses quadrature signals. This means that two signals (channel A and channel B) are used to detect changes in the rotation angle. This detection method increases the encoder's resolution by a factor of four.

The digital resolution is determined by the number of bits used to record the number of pulses. It is calculated as:

$$\Delta\Theta_d = \frac{\Theta_{MAX}}{2^b} = \frac{360°}{2^{12}} = \frac{360°}{4096} = 0.0879°$$

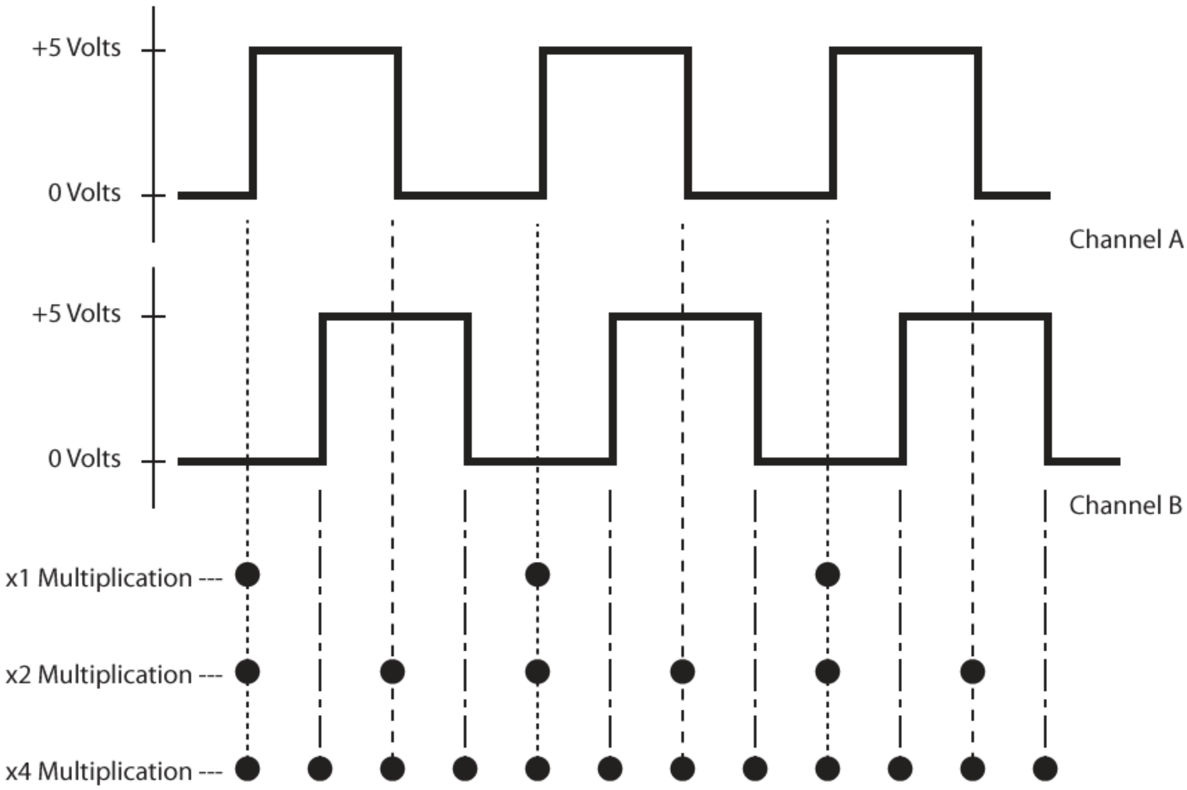
The physical resolution depends on the number of windows on the encoding disk. From the task data, we can see that the encoder has 500 windows per centimetre of the encoding disk. The number of windows is calculated using the equation:

$$N = w \cdot d$$

The equation for calculating the physical resolution is:

$$\Delta\Theta_f = \frac{\Theta_{MAX}}{N}$$

This equation would apply if only one channel were used. However, since two channels are used, one transition of the two sensors across a window on the encoding disk can detect four changes, resulting in four pulses. The sketch below shows the pulse counting options:



The first option is to use only one of the two channels and count pulses at the transition of the signal from 0 to 1. This yields one pulse per channel period. The second option is to use both channels and count pulses at the transition from 0 to 1. This yields two pulses per channel period. The third option is to use both channels and count pulses at both transitions from 0 to 1 and from 1 to 0. This yields four pulses per channel period..

The physical resolution for the last option is:

$$\varDelta\varTheta_f = \frac{\varTheta_{MAX}}{4\ \cdot N}$$

Assuming an ideal encoder, the digital and physical resolutions are equal. This happens if the pulse train is ideal and the transitions between digital pulse levels are infinitely short and infinitely fast. In this case:

$$\varDelta\varTheta_d = \varDelta\varTheta_f$$

$$\frac{\varTheta_{MAX}}{2^b} = \frac{\varTheta_{MAX}}{4 \cdot\ N} = \frac{\varTheta_{MAX}}{4 \cdot\ w \cdot d}$$

From the above equation, we can derive the diameter of the encoding disk:

$$d = \frac{2^{b-2}}{w} = 2.05\ cm$$

Next, we determine the rotation speed of a motor equipped with such an incremental encoder. There are several methods for determining speed, but we choose the pulse counting method. This means counting the pulses produced by the light sensor within a predetermined time. The angular speed of the motor can generally be determined as:

$$\omega = \frac{\varTheta}{T}$$

where $\varTheta$ k is the rotation angle and T is the time unit.

Č If the encoder detects n pulses in time T, the average time of one pulse can be calculated as:

$$T_p = \frac{T}{n}$$

If the encoding disk has N windows and quadrature signals with detection of transitions on both signal sides are used, the motor rotates through an angle in the time of one pulse:

$$\varTheta_p = \frac{\varTheta_{MAX}}{4 \cdot N}$$

Combining both equations, we get the final equation for the motor's angular speed:

$$\omega = \frac{\Theta_p}{T_p} = \frac{\frac{\Theta_{MAX}}{4 \cdot N}}{\frac{T}{n}} = \frac{\Theta_{MAX} \cdot n}{4 \cdot N \cdot T} = 17.57\ \frac{\circ}{s} = 0.307\ \frac{rad}{s} = 0.0488 \frac{rev}{s}$$

Advantages of the incremental encoder:

- lower cost
- higher resolution and accuracy for measuring relative movements
- higher frequencies of output signals
- easier installation

Disadvantages and limitations of the incremental encoder:

- requires a reference signal
- does not provide an absolute rotation angle
- cumulative errors can occur
- requires synchronization for measuring rotations on multiple axes
- not immediately usable after a power outage (requires a reset)