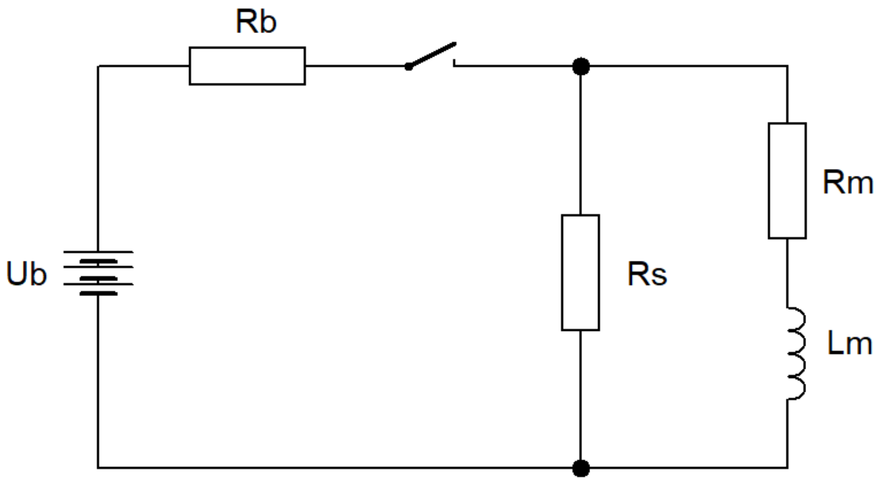
TASK 1

The figure shows a simplified model of a DC electric motor. The switch in the circuit is closed until time $t = 0$, and then we open it. Determine the voltage on the electric motor u_M as a function of time.

Circuit:



Data:

$U_b = 24\,V,\qquad R_b = 2\,\Omega$

$R_m = 1\,\Omega,\qquad L_m = 5\,H$

$R_s = 20\,\Omega$

$u_M(t) = ?$

Solution:

A simplified model of an electric motor includes the properties of its basic elements:

- R_m: the winding resistance, this resistance determines what current flows through the motor at a given voltage and affects losses
- L_m: the inductance of the motor winding, causes a current delay, is related to the energy stored in the magnetic field and affects the dynamic properties of the motor (starting, stopping)
- R_s: the resistance due to other elements or properties of the motor, e.g. losses due to eddy currents, resistance at the contact of brushes and commutator, etc.

To calculate the voltage on the motor $u_m(t)$ means that we have to calculate the voltage in the branch with R_m and L_m. First, let us recall the basic equation for a coil:

$$u = L\frac{di}{dt}$$

The voltage across the coil is proportional to the change in current flowing through the coil winding. When we close the switch and thus enable the movement of electrons (electric current) along the winding of the coil, this current does not immediately reach its final value. Due to the gradual creation of a magnetic field in the coil, the current increases exponentially. At first, the value of the current increases rapidly, but over time the change is smaller and smaller and the current converges towards the final value. The time required for the coil current to reach its final value depends on the time constant:

$$\tau = \frac{L_m}{R}$$

where R is a substitute or Thevenin resistance "seen" by the coil from its connection terminals:

$$R = R_{Th} = R_b||R_s + R_m = 2.818\ \Omega$$

The time constant is therefore:

$$\tau = \frac{L_m}{R} = \frac{5\ H}{2.818\ \Omega} = 1.774\ s$$

In general, we assume that the transient phenomenon ends in about 4 or 5 times the time constant, since at that point the observed value no longer changes significantly. In our case, this means in about 8 seconds.

When the transient ends, DC conditions occur. The current through the coil no longer changes; the voltage on the winding is therefore:

$$u_{Lm} = L_m \frac{di}{dt} = L_m \cdot 0 = 0\ V$$

The current in the circuit after the end of the transient phenomenon is thus determined only by the resistances R_m, R_s and R_s:

$$I = \frac{U_b}{R_b + R_s||R_m} = 8.13\ A$$

The current in the coil winding is then:

$$I_m = I \frac{R_s}{R_s + R_m} = 7.74\ A$$

If we assume that the switch is initially closed for a sufficient time, we can assume that the transient has already gone off. If we then open the switch, we deprive the circuit of the main source of energy. The only source that temporarily causes the electrons to move around the circuit is the energy stored in the coil (in the magnetic field of the coil). This energy is used for the movement of electrons, but because they move along the wires and bump into the atoms in the structure of the wire (in the copper), they lose energy. Their kinetic energy is converted into heat energy. Eventually, all the initial energy is converted into heat and the electrons stop moving. Let us write down the equations for this transient phenomenon:

$$u_{Lm} = u_{Rm} + u_{Rs}$$

$$L_m \frac{di_{Lm}}{dt} = i_{Rm} R_m + u_{Rs} R_s$$

Since it is a simple loop circuit and the currents on all elements are the same, the equation can be simplified:

$$L_m \frac{di}{dt} = iR_m + iR_s$$

We obtain a homogeneous linear differential equation for the current in the circuit. By solving the equation according to the classic procedure with the extension of the solution in exponential form, we would get:

$$i(t) = I_m e^{-\frac{t}{\tau}}$$

where τ is a circuit time constant:

$$\tau = \frac{L_m}{R_m + R_s} = 0.238\ s$$

We found that the transient phenomenon when emptying the coil would be much faster than in the previous step, when we filled the coil.

We also determine the voltage on the coil winding, i.e. the total voltage on R_m and L_m:

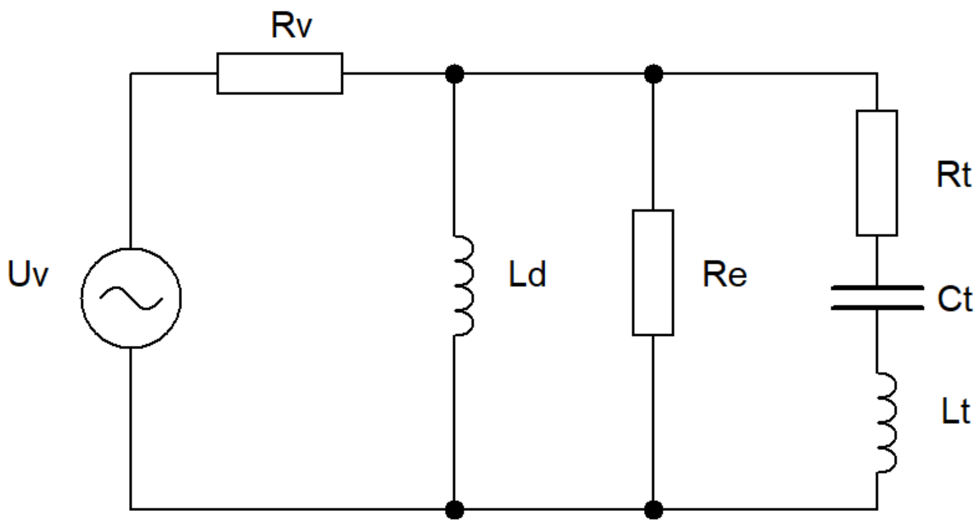
$$u_m(t) = L_m \frac{di}{dt} + iR_m = L_m I_m \left(-\frac{1}{\tau}\right) e^{-\frac{t}{\tau}} + R_m I_m e^{-\frac{t}{\tau}} = I_m \left(R_m - L_m \frac{1}{\tau}\right) e^{-\frac{t}{\tau}} =$$
$$I_m \left(R_m - L_m \frac{1}{\frac{L_m}{R_m + R_s}}\right) e^{-\frac{t}{\tau}} = I_m (R_m - R_m + R_s) e^{-\frac{t}{\tau}} = I_m R_s e^{-\frac{t}{\tau}} = 154.8\ e^{-\frac{t}{0.238}}$$

TASK 2

The diagram shows a simplified model of the input impedance of an electric motor for operation at high frequencies. The inductance L_d represents the non-ideal coupling between the stator and rotor windings; the resistance R_e represents the losses due to Eddy currents in the core of the windings, and the branch with the connection $R_t - L_t - C_t$ represents the parasitic capacitance between the winding turns at high frequencies.

Determine the equivalent impedance of the circuit and the amplitudes of the currents through the individual elements.

Circuit:



Data:

$U_S = 100 \measuredangle 0° \, V$, $R_S = 50 \, \Omega$, $f = 100 \, kHz$

$L_d = 5.1 \, m\Omega$, $R_e = 3.9 \, k\Omega$

$R_t = 324 \, \Omega$, $L_t = 0.27 \, mH$, $C_t = 29 \, pF$

$Z_{nad}, I_{R_v V}, I_{L_d V}, I_{R_e V}, I_{R_t V}, =?$

Solution:

This slightly more detailed model enables a better understanding and analysis of the operation of electric motors at high frequencies, where completely simplified models, such as the one in task 1, do not take into account all relevant non-ideal properties. It can be used to design motors with better efficiency and lower losses when operating at high frequencies.

In this model, there are three main components: L_d, R_e, and $R_t - C_t - L_t$ branch, each representing a specific non-ideal assembly or loss in the motor::

- L_d (non-ideal coupling inductance between the rotor and stator windings): represents the inductance arising from the magnetic coupling between the rotor and stator windings. In an ideal motor, this assembly would allow perfect power transfer without losses, but in reality, some losses occur due to an imperfect magnetic assembly. This includes losses due to magnetic field dissipation and imperfect energy transfer between the windings.
- R_e (winding core eddy current losses): represents the resistance that models the winding core eddy current losses. Eddy currents are induced currents in conductive materials exposed to a changing magnetic field, typically found in the cores of electric motor windings. These currents cause losses in the form of heat, which reduces the efficiency of the motor. At high frequencies, the effects of eddy currents increase, causing even greater losses.
- $R_t - C_t - L_t$ branch (parasitic capacitance between winding turns at high frequencies): this branch of the model represents a combination of resistance (R_t), capacitance (C_t) and inductance (L_t), which together model the effect of parasitic capacitance between winding turns. At high frequencies, there is a greater influence of the capacitive connections between the threads, which can lead to unwanted resonance phenomena and additional losses.

The electric motor model is powered by alternating voltage. We are looking for an alternate circuit impedance. For the calculation, we will use the table of impedances and admittances for the basic components, the resistor, the coil and the capacitor:

	impedanca Z $[\Omega]$	**admitanca Y $[S]$**
upor R $[\Omega]$	R	G
kondenzator C $[F]$	$-j\,X_C = -j\,\dfrac{1}{\omega C}$	$jB_C = j\omega C$
tuljava L $[H]$	$j\,X_L = j\omega L$	$-jB_L = -j\,\dfrac{1}{\omega L}$

First, let us take a good look at the circuit and all the connections. On the right, we have three parallel branches, one with inductance L_d, one with resistance R_e and the third with connection $R_t - C_t - L_t$. On the left, the resistance R_v is also connected in series to the three parallel branches.

Let us start the solution by calculating the impedance of the rightmost branch:

$$Z_t = R_t + jX_{Lt} - jX_{Ct} = R_t + j\omega L_t - j\frac{1}{\omega C_t} = 324\,\Omega + j\,169.6\,\Omega - j\,54.88\,k\Omega =$$
$$= 324\,\Omega - j\,54.711\,k\Omega$$

We will assemble the three parallel branches together by determining their admittances and summing them. The admittance of the rightmost branch is obtained by the inverse of the impedance:

$$Y_t = \frac{1}{Z_t} = \frac{1}{324 - j54711} = \frac{(324 + j\,54711)}{(324 - j\,54711)(324 + j\,54711)} = \frac{324 + j\,54711}{324^2 + j54711^2}$$
$$\doteq 0 + j\frac{1}{54711} = j\,1.83\,10^{-5}\,S$$

We determine the total admittance of three parallel branches:

$$Y = Y_{Ld} + Y_{Re} + Y_t = -jB_{Ld} + \frac{1}{R_e} + Y_t == -j\frac{1}{\omega L_d} + \frac{1}{R_e} + Y_t$$
$$= -j\,3.121\,10^{-4} + 2.564\,10^{-4} + j\,1.83\,10^{-5}\,S$$
$$= 2.564\,10^{-4} - j\,2.938\,10^{-4}\,S$$

To the trio of parallel branches, we must successively add the resistance R_v. In series connections, the elements are put together by adding their impedances. Therefore, we first convert the total admittance of the parallel branches into impedance:

$$Y = 2.564\,10^{-4} - j\,2.938\,10^{-4}\,S = 0.3896\,10^{-3} \measuredangle -48.93\,°$$

$$Z = \frac{1}{Y} = \frac{1}{0.3896\,10^{-4} \measuredangle -48.93\,°} = 2567 \measuredangle 48.93\,° = 1686.2 + j\,1935.2\,\Omega$$

Finally, we can calculate the equivalent impedance of the circuit:

$$Z_{NAD} = Z_{Rv} + Z = 50 + 1686.2 + j\,1935.2\,\Omega = 1736.2 + j\,1935.2\,\Omega$$
$$= 2600 \measuredangle 48.1\,°\,\Omega$$

Using the equivalent impedance, we can calculate the current flowing from the voltage source:

$$I = \frac{U}{Z_{NAD}} = \frac{100 \measuredangle 0\,°\,V}{2600 \measuredangle 48.1\,\Omega} = 38.5\,10^{-3} \measuredangle -48.1\,°\,A = 38.5 \measuredangle -48.1\,°\,mA$$

This current flows from the voltage source and across the resistor R_v. The amplitude of the current across this resistor is equal to the amplitude of the total current:

$$I_{R_v V} = I = 38.5\ mA$$

The current then splits at the node into currents across three parallel branches. The size of the currents is determined by current dividers, where the admittances of the parallel branches, which we have already determined, are used in the equation:

$$I_{L_d V} = I_V \frac{Y_{Ld}}{Y_{Ld} + Y_{Re} + Y_t} = I_V \frac{B_{Ld}}{Y_{Ld} + Y_{Re} + Y_t} = 38.5\ mA \frac{3.121\ 10^{-4}}{0.3896\ 10^{-3}} = 30.84\ mA$$

$$I_{R_e V} = I_V \frac{Y_{Re}}{Y_{Ld} + Y_{Re} + Y_t} = I_V \frac{\frac{1}{R_e}}{Y_{Ld} + Y_{Re} + Y_t} = 38.5\ mA \frac{2.564\ 10^{-4}}{0.3896\ 10^{-3}} = 25.34\ mA$$

$$I_{L_d V} = I_V \frac{Y_t}{Y_{Ld} + Y_{Re} + Y_t} = 38.5\ mA \frac{1.83\ 10^{-5}}{0.3896\ 10^{-3}} = 1.81\ mA$$

TASK 3

We connect an $1\ kW$ electric motor to an AC voltage source with an RMS value of $U_{RMS} = 230\ V$ and a frequency of $f = 50\ Hz$. Through measurements, we find that the system has a power factor of 0.8.

To correct the power factor, we connect a capacitor in parallel to the motor. Calculate the size of the capacitor needed to correct the power factor to 0.95. Calculate also the current flowing from the source in connection without and with the capacitor.

Data:
$U_{RMS} = 230\ V$
$f = 50\ Hz$
$PF = 0.8$

$C\ for\ PF = 0.95 = ?\ , I_{noncomp}, I_{comp} = ?$

Solution:

In circuits powered by alternating current sources, we refer to different types of power consumed in the circuit. The part of the power that results, on average, in a net transfer of energy in one direction (from the source to the load) is called real power (also known as active power). This is the power consumed by resistive loads (anything that has electrical resistance). The part of the power that does not result in a net transfer of energy in one direction, but oscillates between the source and the load, is called reactive power. These oscillations occur due to the presence of capacitors and inductors, which have the ability to store energy and therefore act as secondary sources. The combination of active power and reactive power, taken as a vector sum, is known as apparent power.

The apparent power on the load is calculated by multiplying the effective value of the voltage across the load by the effective value of the current through the load

$$P_N = U_{ef} I_{ef}$$

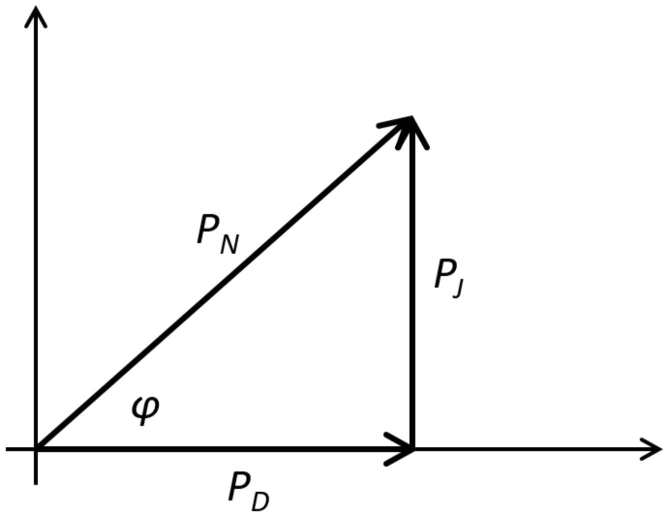
The real power consumed by the load is calculated from the apparent power, taking into account the phase shift φ between the voltage and current signals. φ is the angle by which the voltage leads or lags the current in phase. The real power is calculated as

$$P_D = U_{ef} I_{ef} \cos \varphi$$

The reactive power is calculated as

$P_J = U_{ef} I_{ef} \sin \varphi$

All three types of power can be represented in the characteristic power triangle. On the x-axis, we draw the real power (active power), on the y-axis, we draw the reactive power, and the apparent power is obtained by the vector sum of these two components.



Since reactive power is the part of the power that only oscillates between the source and the load and cannot be usefully used (e.g., for conversion into heat, mechanical work, etc.), it represents losses for the system. This power flows between the source and the load continuously, so larger industrial consumers need to manage it or pay electricity suppliers for it. The vast majority of reactive power in power systems is generated by asynchronous motors, followed by transformers, with a smaller amount generated by electromagnetic generators and alternators.

The power factor is a quantity that indicates the proportion of reactive power compared to real power or apparent power. The higher the reactive power, the lower the power factor. In an ideal case, when the load does not produce reactive power, the power factor is equal to 1. The power factor is calculated using the equation

$$PF = \frac{P_D}{P_N} = \cos \varphi$$

The power factor is therefore equal to the cosine of the angle between the real power vector and the apparent power vector in the power triangle. It is also the phase angle between the voltage and current signals on the load.

Given that the initial power factor is 0.8 and the load is a 1 kW electric motor, we know that 1 kW of real power is consumed by the motor. Using this information, we can determine the apparent power

$$P_N = \frac{P_D}{\cos \varphi} = \frac{1000}{0.8} = 1250\,VA$$

We supply the motor with an alternating voltage source, which has an effective value of 230 V. Using the equation for apparent power, we calculate the magnitude of the current

$$I_{ef} = \frac{P_N}{U_{ef}} = \frac{1250\,VA}{230\,V} = 5.43\,A$$

Since we want to change the power factor, we will add a capacitor to the circuit to achieve correction or compensation. The capacitor will affect the reactive power, reducing the inductive reactive power contributed by the motor windings in the rotor and/or stator.

The initial reactive power is mostly inductive in nature. We can determine the reactive power using the Pythagorean theorem, which relates the apparent power, real power, and reactive power in a right triangle

$$P_J = \sqrt{P_N^2 - P_D^2} = 750\,VAR$$

To estimate the inductance of the windings in the motor, we need to understand the relationship between the reactive power, the current, and the impedance of the winding. Reactive power in an inductive load can be expressed in terms of inductance, current, and frequency

$$P_J = I_{ef}^2\,X_L = I_{ef}^2\,\omega L$$

The inductance of the winding is thus

$$L = \frac{P_{JL}}{I_{ef}^2\,\omega} = \frac{P_{JL}}{I_{ef}^2\,2\pi f} = 80.97\,mH$$

This is the initial state before attempting to correct the power factor. The aim of the correction is to increase the power factor to a value of 0.95. Let us repeat the previous calculations for the target power factor. At the same operating power, (the motor remains the same, consuming 1 kW of operating power), the new apparent power will be equal to:

$$P_{N2} = \frac{P_D}{\cos\varphi} = \frac{1000}{0.95} = 1052.6\,VA$$

Reactive power after correction will be equal to

$$P_{J2} = \sqrt{P_{N2}^2 - P_D^2} = 328.7\,VAR$$

This reactive power consists of the initial reactive power P_J and the reactive power due to the added capacitor P_{JC}. From the properties of the capacitor, we know that it always has the opposite effect of the inductor (winding). Assuming that the initial reactive power is caused by the winding, the reactive power due to the capacitor is subtracted from the initial reactive power. Therefore, the reactive power of the capacitor is

$$P_{JC} = P_{J2} - P_J = 421.3\,VAR$$

To calculate the appropriate capacitor, we rewrite the equation for power, but now we use the impedance (reactance) of the capacitor in the equation

$$P_{JC} = \frac{U_{ef}^2}{X_C} = \frac{U_{ef}^2}{\frac{1}{\omega C}} = U_{ef}^2 \omega C$$

The capacitor that is added in parallel with the motor to improve the power factor to a value of 0.95 must therefore have a capacitance of

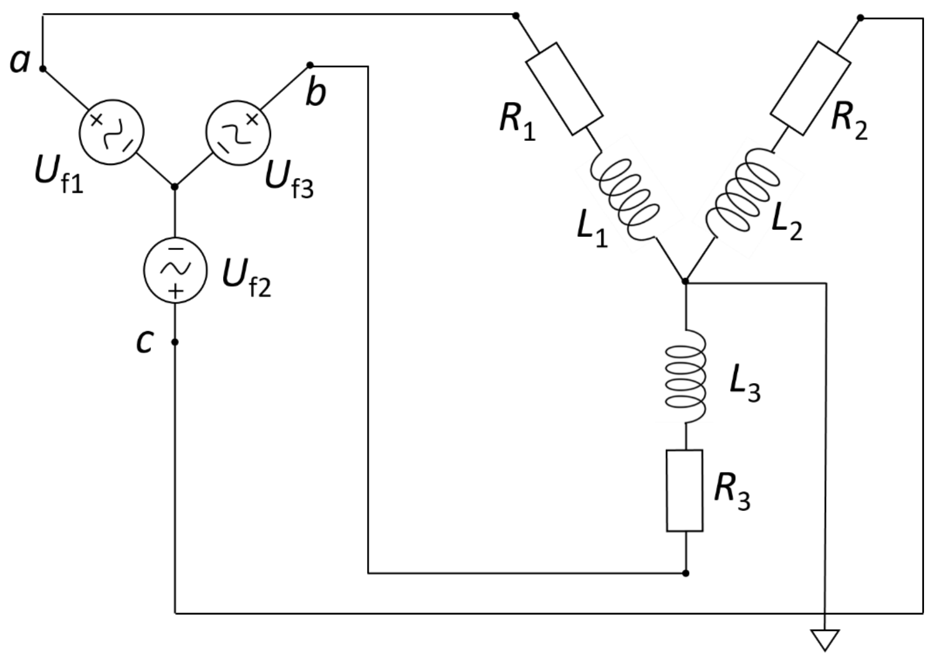
$$C = \frac{P_{JC}}{U_{ef}^2 \omega} = 25.4\,\mu F$$

TASK 4

The 3-phase star-connected electric motor is powered by a 3-phase voltage source, where individual AC sources are also star-connected. The inductive loads representing the motor are the same in all phases. Determine the currents across all motor windings and the current in the ground conductor.

How much power does the motor receive? What is the power factor of the motor? Why do we not usually ground these types of motors in the industry?

Circuit:



Data:

$U_{ph1} = 230 \measuredangle -30° \, V$

$U_{ph2} = 230 \measuredangle 90° \, V$

$U_{ph3} = 230 \measuredangle 210° \, V$

$f = 50 \, Hz$

$R_1 = R_2 = R_3 = 40 \Omega$

$L_1 = L_2 = L_3 = 5 \, m\Omega$

$I_{L1}, I_{L2}, I_{L3} = ?$
$P_M, PF = ?$

Solution:

In 3-phase voltage sources, we use three alternating voltage sources in a certain arrangement. On the load side, we also have three components of the load, which are in a certain connection. Both 3-phase sources and 3-phase loads are typically in one of the two possible connections:

- star connection: branches with sources (or loads) are connected together with one connection in the central node (the centre of the star)
- delta connection: branches with sources (or loads) are connected one after the other; the end of the first branch is connected to the beginning of the second branch, the end of the second branch to the beginning of the third branch, etc.

Three phase sources (generators) are usually star-connected, while loads can be either star-connected or delta-connected.

One could wonder why we should use 3-phase systems at all, or what the advantages of 3-phase systems over 1-phase systems are. Some of the advantages are:

- 3 phase systems provide constant power to the motor (load)
- for similar motor power in 3-phase systems, the motors can be smaller (space and cost saving)
- 3 phase systems are less noisy as the power is constant, unlike 1 phase systems where the power oscillates, so there is more vibration and noise
- 1-phase motors can be connected to a 3-phase generator, if only one of the phases is used
- savings in conductor material:
    - with a 1-phase system, we need two conductors, one conductor (phase conductor) carries current towards the load, and the other conductor (neutral conductor) carries current back to the source
    - in the case of a 3-phase system, we need one conductor for each phase, while the neutral conductor can be shared → we therefore need four conductors in total
    - since the same maximum current flows through all phase conductors, the phase conductors can be of the same diameter → the diameter of the wire determines the maximum possible current
    - o in the special case, when the 3-phase load is symmetrical (the phase impedances are the same in all three phases), the neutral conductor is not needed at all, since the sum of the currents in the centre of the star is equal to 0 due to the symmetry
- no need for additional starting mechanisms:

- with a single-phase motor, we need an additional starting mechanism, since the motor cannot start by itself
    - when AC voltage is applied to the motor, a current flows through the stator winding and a magnetic field is created
    - alternating excitation constantly changes sign, therefore the magnetic field on the stator also changes direction (N-S or S-N)
    - these two components of the magnetic field are in antiphase and oppose each other
    - a voltage is induced on the rotor winding due to the changing magnetic field of the stator, which opposes the changes in the magnetic field → the induced voltage drives a current that creates another magnetic field
    - for motor rotation or to create torque, we need two magnetic fields that are offset by 90°
    - the problem is that with a single-phase motor, the magnetic field of the stator constantly changes direction ⍰ one component of the field would rotate the motor in one direction and the other in the opposite direction, so the motor does not move
    - we need some mechanism that creates a 90° phase shift; the options are:
        - starting capacitor
        - additional starting winding
        - induction motor with shaded poles
- with a 3-phase system, we have a generator consisting of three alternating current sources, whose voltages are offset by 120°
    - this creates a rotating magnetic field of the generator which is also transferred to the motor
    - since the voltages of the individual sources (phase voltages) are each connected to one winding of the motor, we get voltages on the windings of the motor that rotate all the time, which results in phase lags that create torque and rotate the motor

A symmetrical 3-phase load is a motor with three windings placed in a star connection and having the same impedance:

$$Z_m = R_m + jX_m$$

Let us write the voltages on the individual phases (impedances) of the 3-phase motor. For correct notation, we need to carefully examine the circuit diagram and check which part of the generator each load impedance is connected to. We find that the phases of the motor are connected to the individual phases of the generator. The neutral conductor is not shown in the diagram since it is a symmetrical load and this conductor is not needed. However, we know that the voltage on impedance Z_1 is equal to the voltage of source 1 (phase voltage U_{f1}), the voltage on impedance Z_2 is equal to the phase voltage U_{f2}, and the voltage on impedance Z_3 is equal to the phase voltage U_{f3}.

$$U_{Z1} = U_{an} = U_{f1} = 230 \measuredangle -30° \, V$$

$$U_{Z2} = U_{bn} = U_{f2} = 230 \measuredangle 120° \, V$$

$$U_{Z3} = U_{cn} = U_{f3} = 230 \measuredangle 210° \, V$$

When writing the voltage on the load, it is also necessary to take into account the location of the sources on the generator. In the diagram, we notice that all sources are marked with + and - signs. These are not markings as we know from DC sources, but rather they indicate the phase lags of the voltages on the sources. The data on the phase angles of the source voltages are valid only if the sources are connected in a star configuration at the - connector. If the connections of one of the sources were changed, it would mean that it works in anti-phase and the voltage would be offset by 180°.

From the voltage, let us calculate the currents in all phases of the load. Due to symmetry, the load impedances are the same:

$$Z_{m1} = Z_{m2} = Z_{m3} = R_m + jX_m = 40 + j\,1.57 \, \Omega = 40.031 \measuredangle 2.25° \, \Omega$$

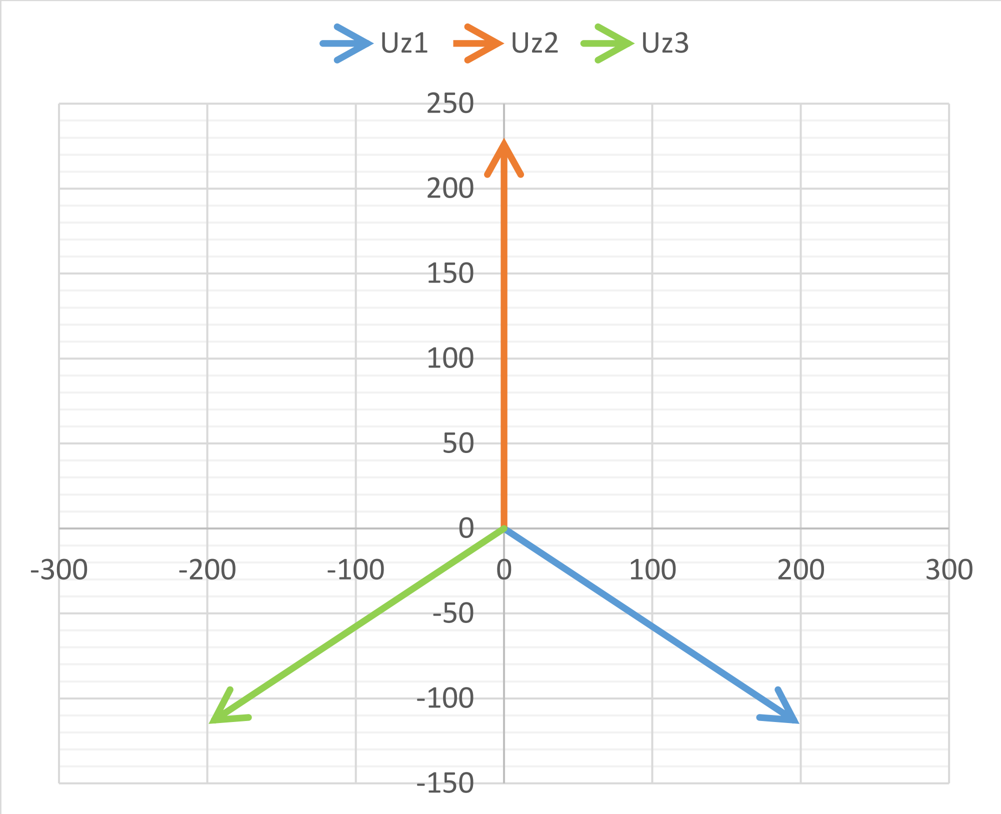
The currents are therefore:

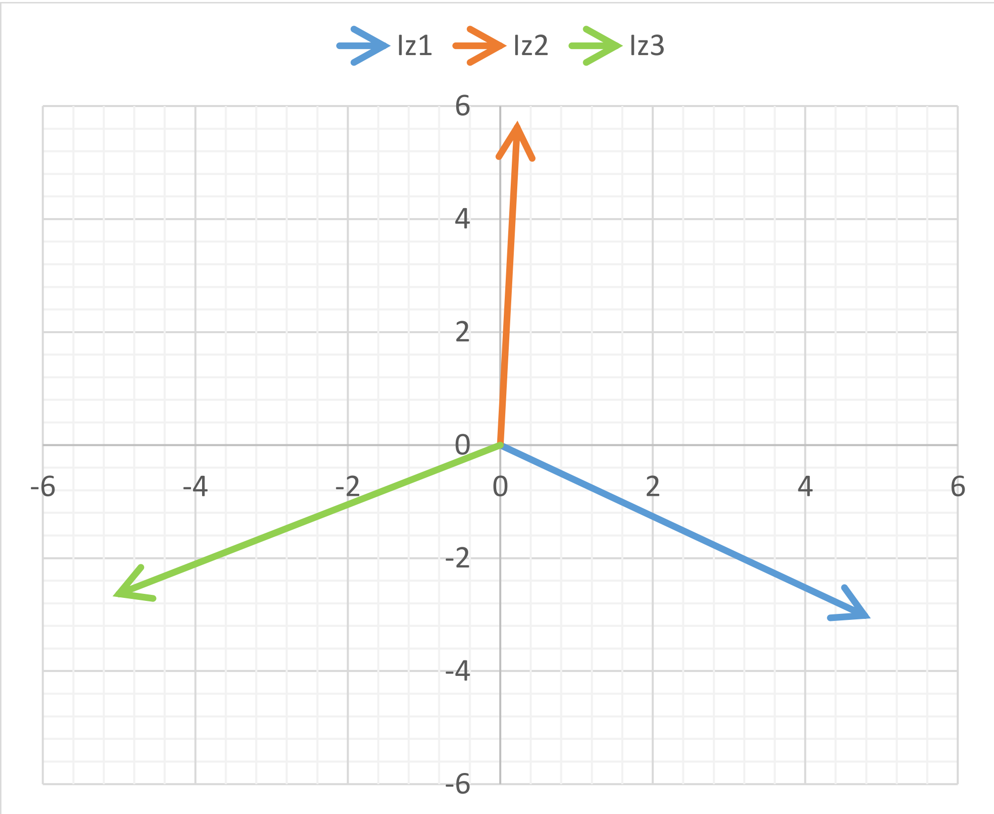
$$I_{Z1} = \frac{U_{Z1}}{Z_{m1}} = \frac{230 \measuredangle -30°}{40.031 \measuredangle 2.25° \, \Omega} = 5.746 \measuredangle -32.25° \, A$$

$$I_{Z2} = \frac{U_{Z2}}{Z_{m2}} = \frac{230 \measuredangle 120°}{40.031 \measuredangle 2.25° \, \Omega} = 5.746 \measuredangle 87.75° \, A$$

$$I_{Z3} = \frac{U_{Z3}}{Z_{m3}} = \frac{230 \measuredangle 210°}{40.031 \measuredangle 2.25° \, \Omega} = 5.746 \measuredangle 207.75° \, A$$

These are the currents flowing into the node at the centre of the star connection of the impedances of the 3 phase motor. Let us draw a phasor diagram for voltages and currents:





If the load is symmetrical, the phasor diagram for currents is similar to the phasor diagram for voltages, except that it is phase-shifted. However, if the loads were not symmetrical, the phasor diagram for the currents would be quite different.

Let us check the total current at the node in the centre of the star:

$$I = I_{Z1} + I_{Z2} + I_{Z3} = 5.75 \measuredangle -32.2° + 5.75 \measuredangle 87.8° + 5.75 \measuredangle 207.8°$$
$$= 4.863 - j\,3.068 + 0.22 + j\,5.75 - 5.08 - j\,2.68 = 0\,A$$

We come to an important conclusion: the total current into the centre node of the star load is 0 A. This means that for a symmetrical 3-phase load, we do not need a neutral conductor; only three conductors are sufficient. While we get three times more power with a 3-phase load, we only need 1.5 times more conductors (copper) for this configuration.

If the load is not symmetrical, the sum of the currents at the centre node will not be 0 A. However, this sum will be relatively small compared to the currents in the motor windings. For a small current, a wire with significantly smaller thickness than that required by the phase conductors, which need to withstand currents of almost 6 A, will be sufficient.

Let us also calculate the real power of the motor and the power factor. The real powers on the individual phases of the motor are:

$$P_{D1} = U_{Z1ef} I_{Z1ef} \cos\varphi_{Z1} = 1320.46\,W$$

$$P_{D2} = U_{Z2ef} I_{Z2ef} \cos\varphi_{Z2} = 1320.46\,W$$

$$P_{D3} = U_{Z3ef} I_{Z3ef} \cos\varphi_{Z1} = 1320.46\,W$$

The total real power obtained by the motor is thus:

$$P_D = P_{D1} + P_{D2} + P_{D2} = 3961.4\,W$$

Apparent powers are calculated in a similar way to real powers:

$$P_{N1} = U_{Z1ef} I_{Z1ef} = 1321.48\,VA$$

$$P_{N2} = U_{Z2ef} I_{Z2ef} = 1321.48\,VA$$

$$P_{N3} = U_{Z3ef} I_{Z3ef} = 1321.48\,VA$$

The total apparent power is thus:

$$P_N = P_{N1} + P_{N2} + P_{N2} = 3964.4\,W$$

The power factor is:

$$PF = \frac{P_D}{P_N} = \frac{3961.4\,W}{3964.4\,W} = 0.99923$$

The result is expected since the inductive component of the phase impedances of the load is significantly smaller than the resistive component.

Finally, let us explain why motors are usually not directly grounded in industry, i.e., why there is no grounding conductor on the motor. The grounding conductor is intended for protection in case of damage, improper connection, etc., where an additional (unwanted) current appears and is directed into the ground.

If we have multiple motors in a 3-phase system, each with its own ground conductor connected to the centre of its star connection, this creates many possible paths for resulting currents to flow in the event of a problem. Therefore, we must protect individual parts of the system with circuit breakers that prevent current from flowing into other parts of the system. This complicates the system design significantly, and with poor design, it can happen that disconnect relays break the connection at the wrong place.

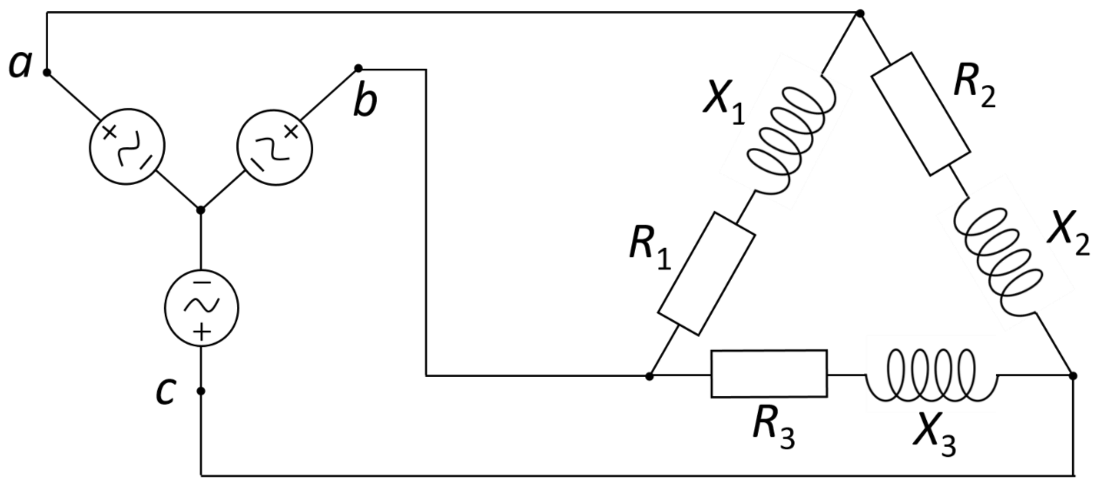
The solution lies in arranging common grounding at the source. All unwanted currents flow into this single grounding conductor, and protection for unwanted currents to other parts of the system is arranged with circuit breakers only at the source..

TASK 5

Determine the apparent and real power delivered to a 3-phase R_L load connected in a delta configuration by a 3-phase AC source connected in a star configuration. Additionally, calculate the power factor of this system.

Voltages are given in RMS values.

Circuit:



Data:

$U_{ph1} = 230 \measuredangle 0° \, V$

$U_{ph2} = 230 \measuredangle 120° \, V$

$U_{ph3} = 230 \measuredangle 240° \, V$

$R_1 = R_2 = R_3 = 40 \, \Omega$

$X_1 = X_2 = X_3 = 60 \, \Omega$

$P_T, P_A, PF = ?$

Solution:

The task differs from the previous one mainly in that the load impedances are connected in a delta connection this time. This connection is already visually quite different from the star connection, but it also behaves differently functionally. The essential thing to note from the schematic is the connection of the individual load impedances to the generator. We understand that impedances are connected between two phase voltages:

- impedance Z_1 is connected between terminals a and b, i.e. between phases 1 and 2

- impedance Z_2 is connected between terminals a and c, i.e. between phases 1 and 3
- impedance Z_3 is connected between terminals b and c, i.e. between phases 2 and 3

In the case of a delta connection of a 3-phase load (and in the case of a star connection of a 3-phase generator), the individual load impedances give the so-called line voltages. Let's first calculate the voltage across the impedance Z_1:

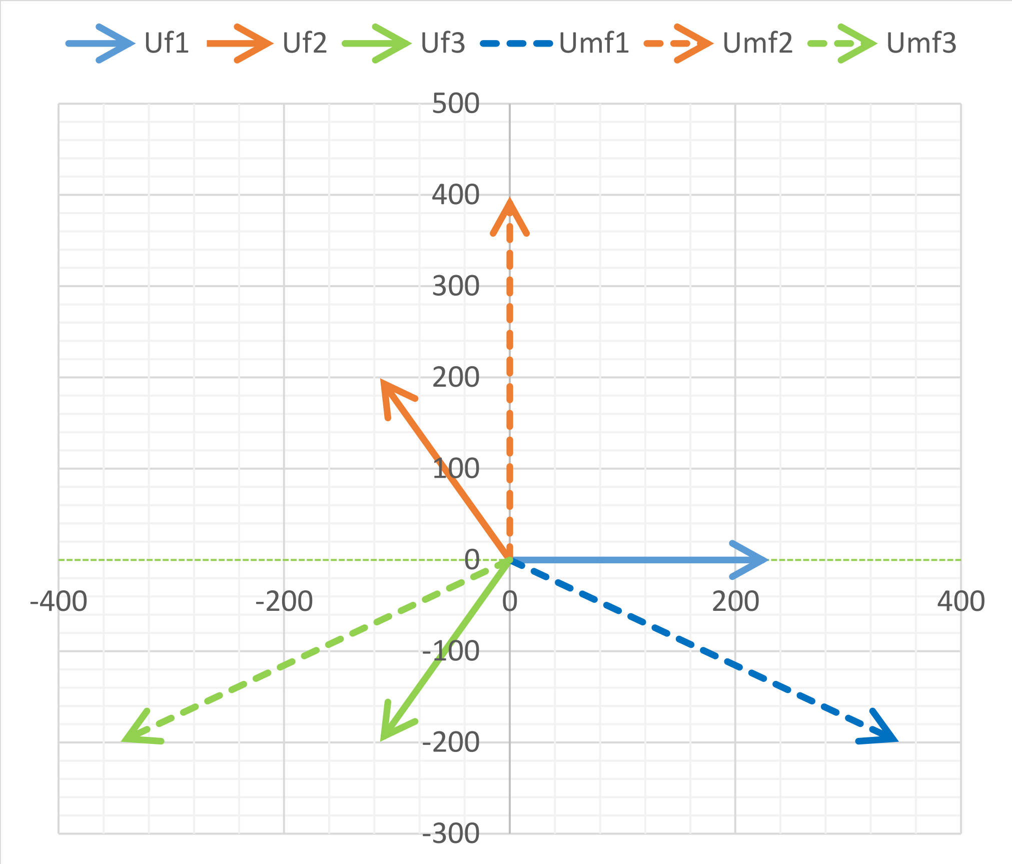
$$\begin{aligned} U_{Z1} = U_{an} - U_{bn} &= 230 \measuredangle 0° \, V - 230 \measuredangle 120° \, V \\ &= 230 + j\,0\,V - (-115 + j\,199.19\,V) = 345 - j\,199.19 \\ &= 398.36 \measuredangle -30° \, V \end{aligned}$$

If we compare the above result, we find that the line voltage is higher by a factor of $\sqrt{3}$ according to the effective value, and the phase is shifted by 30°. A similar finding is also made for the other line voltages:

$$\begin{aligned} U_{Z2} = U_{bn} - U_{cn} &= 230 \measuredangle 120° \, V - 230 \measuredangle 240° \, V \\ &= -115 + j\,199.19\,V - (-115 \; - j\,199.19\,V) = 0 + j\,398.36 \\ &= 398.36 \measuredangle 90° \, V \end{aligned}$$

$$\begin{aligned} U_{Z3} = U_{cn} - U_{an} &= 230 \measuredangle 240° \, V - 230 \measuredangle 0° \, V \\ &= -115 \; - j\,199.19\,V - (230 + j\,0\,V) = -345 - j\,199.19 \\ &= 398.36 \measuredangle 210° \, V \end{aligned}$$

We therefore get voltages on all impedances that are $\sqrt{3}$ times higher than the phase voltages and offset by 30°. Let us look at the phase and line voltages in the phasor diagram:



Let us also determine the currents on the load impedances. First, let us determine the impedances in polar form:

$$Z = Z_{m1} = Z_{m2} = Z_{m3} = 40 + j\,60\,\Omega = 72.11 \measuredangle 56.31\,°\,\Omega$$

$$I_{Z1} = \frac{U_{Z1}}{Z_{m1}} = \frac{398.36 \measuredangle 0°\,V}{72.11 \measuredangle 56.31\,°\,\Omega} = 5.52 \measuredangle -56.31°\,A$$

$$I_{Z2} = \frac{U_{Z2}}{Z_{m2}} = \frac{398.36 \measuredangle 120°\,V}{72.11 \measuredangle 56.31\,°\,\Omega} = 5.52 \measuredangle 63.69°\,A$$

$$I_{Z3} = \frac{U_{Z3}}{Z_{m3}} = \frac{398.36 \measuredangle 240°\,V}{72.11 \measuredangle 56.31\,°\,\Omega} = 5.52 \measuredangle 183.69\,°\,A$$

Let us calculate the real power at the individual load impedances and the total real power:

$$P_{D1} = U_{Z1ef} I_{Z1ef} \cos\varphi_{Z1} = 1220.8\,W$$

$$P_{D2} = U_{Z2ef} I_{Z2ef} \cos\varphi_{Z2} = 1220.8\,W$$

$$P_{D3} = U_{Z3ef} I_{Z3ef} \cos\varphi_{Z3} = 1220.8\,W$$

The total real power received by the motor is thus:

$$P_D = P_{D1} + P_{D2} + P_{D3} = 3662.3\ W$$

Let us calculate the apparent powers:

$$P_{N1} = U_{Z1ef} I_{Z1ef} = 2200.73\ VA$$

$$P_{N2} = U_{Z2ef} I_{Z2ef} = 2200.73\ VA$$

$$P_{N3} = U_{Z3ef} I_{Z3ef} = 2200.73\ VA$$

The total apparent power is thus:

$$P_N = P_{N1} + P_{N2} + P_{N3} = 6602.32\ VA$$

The power factor is:

$$PF = \frac{P_D}{P_N} = \frac{3662.3\ W}{6602.32\ VA} = 0.5547$$

The power factor is equal in magnitude to the phase angle of the individual load impedance. Since the imaginary component of the load is larger than the real one, the power factor is quite small, so considerable reactive power would be consumed on such a load.

Connecting a 3-phase load in a delta or star connection has its advantages and disadvantages. Let us list some:

- star connection:
  - advantages:
    - due to lower voltages on the load impedances and lower currents on the phase conductors, conductors with a smaller diameter and less insulation can be used
    - the problem of asymmetric loads is easier to deal with due to the possibility of using a neutral conductor
    - such connections are suitable for transmission over longer distances (distribution power network)
    - in the European power network, such connections are mostly used for the supply of electricity to households
  - disadvantages:
    - if the load is not symmetrical, we need a neutral conductor in addition to the 3 phase conductors

        - due to the lower voltage on the load impedances (e.g. motor winding) we get a lower output power of the motor, the motor thus rotates at a lower speed
        - such connections are more suitable for applications where high starting torque is not required (smaller loads)
- delta connection:
    - advantages:
        - for all types of loads (symmetrical or asymmetric), three conductors are sufficient, we do not need a neutral conductor
        - on load impedances (for example on the motor winding) we get $\sqrt{3}$ times higher voltage and $\sqrt{3}$ times higher current
        - with delta connection, we get a higher output power at the load impedances (e.g. on the motor), the motor therefore rotates at a higher speed
        - delta connection is suitable for applications where high starting torque is needed (larger motors in industry)
        - with the same voltages of the 3-phase source and with the same load, we get 3 times more power on the load
        - delta connection is mostly used in industry
    - disadvantages:
        - due to larger currents, we need conductors with a larger diameter and with thicker insulation, so such systems are used for power transmission over shorter distances (transmission power network)
        - since it does not have a neutral conductor, it is more difficult to manage unbalanced loads