TASK 1

The DC electric motor develops a torque of $30\ Nm$ during operation. Determine the torque if the current in the armature winding increases by 50% and the magnetic flux in the armature decreases by 10%.

Data:
$T_S = 30\ Nm$
$\Delta I_a = +\ 30\%$
$\Delta \Phi_a = -\ 10\%$

$T_F = ?$

Solution:

Let's start solving the problem by reviewing the basic equations of a DC motor. The current through the stator winding i_f generates a stator magnetic field. The magnetic flux of one pole of the stator winding is denoted by Φ_f. The current i_a flows through the rotor winding. Since the rotor winding is in the magnetic field of the stator, it experiences a magnetic force that causes it to rotate. Using a commutator ensures that the current through the rotor winding always flows in the proper direction so that the force on the winding acts in the same direction and the rotor turns.

Since the magnetic flux of the stator magnetic field through the rotor winding changes (due to rotation), a voltage is induced in the rotor winding:

$$U_i = K\Phi_f\omega_M$$

where:

- $K = \frac{P\ Z}{2\ \pi\ a}$
- P = number of poles of the stator winding
- Z = number of conductors in the rotor slots
- a = number of parallel branches of the rotor winding (a = 2 for wave winding, a = P for lap winding)
- Φ_f = magnetic flux of one pole of the stator
- ω_M = angular speed of the motor

If the magnetic flux of the stator is constant, the induced voltage depends only on the speed:

$$U_i = K_b\omega_M$$

where K_b is the voltage constant. The magnetic flux of one pole of the stator can be determined as:

$$\Phi_f = \frac{N_f I_f}{\mathcal{R}_m}$$

where:

- N_f = number of turns of one pole of the stator
- I_f = current through the stator
- $\mathcal{R}_m$ = reluctance of the magnetic flux path

The voltage constant can be expressed as:

$$K_b = \frac{K\ N_f I_f}{\mathcal{R}_m} = M I_f$$

In the above equation, MM represents the mutual inductance between the stator and rotor windings.

Considering these derivations, the induced voltage can be written as:

$$U_i = M I_f \omega_M$$

This equation means that the induced voltage in the rotor winding is proportional to the current in the stator winding and the motor's rotational speed.

The induced voltage has significant effects on the operation of a DC motor:

- Limits the rotor current: The voltage's sign opposes the change in magnetic flux that caused the induction, reducing the rotor current.
- Speed regulation: The induced voltage is proportional to the motor speed, if the speed increases, the induced voltage increases, subtracting from the supply voltage and thereby reducing the rotor current, which causes the speed to decrease.
- Increased efficiency: Reducing the current also reduces thermal losses in the winding.
- Motor protection: If the motor is suddenly unloaded or the load is completely disconnected, the motor could instantly rotate very quickly, but the induced voltage reduces the current and rotational speed.

If we represent the rotor winding with an equivalent circuit, we can write the voltage equation:

$$U_a = U_i + R_a i_a + L\frac{di_a}{dt}$$

In steady-state, the transient effect on the winding is nullified, and we get:

$$U_a = U_i + R_a I_a$$

From this equation, if we calculate the power of the components of the rotor winding, we get:

- $U_a I_a$ = power supplied from the DC source
- $R_a I_a^2$ = thermal losses in the winding conductor
- $U_i I_a$ = power converted to mechanical power and represents the utilized supplied power

$U_i I_a$ is the part of the supplied electrical power that the system uses to rotate the motor. Considering the power equation in connection with torque and angular speed, we get:

$$P_a = \omega_M M_e = U_i I_a$$

The rotor torque is then:

$$M_e = \frac{U_i I_a}{\omega_M} = K_b I_a$$

Thus, the motor torque is proportional to the current in the rotor and the torque constant K_b. This constant captures the key properties of the stator and rotor:

$$K_b = \frac{P\ Z}{2\ \pi\ a}\frac{N_f I_f}{\mathcal{R}_m}$$

Now let's return to solving the problem. The initial torque developed by the motor is 30 Nm. Then, the armature current (rotor winding current) increases by half, and the magnetic flux of the stator magnetic field through the rotor winding decreases by 10%. Using the above equations to calculate the change in motor torque.

$$M_{e1} = K_{b1} I_{a1} = K\Phi_{f1} I_{a1}$$

$$M_{e2} = K\Phi_{f2} I_{a2} = K(1-0.1)\Phi_{f1}(1+0.5)I_{a1} = K \cdot 0.9 \cdot \Phi_{f1} \cdot 1.5 \cdot I_{a1}$$
$$= 1{,}35\ K\Phi_{f1} I_{a1}$$

$$\frac{M_{e2}}{M_{e1}} = \frac{1.35\ K\Phi_{f1} I_{a1}}{K\Phi_{f1} I_{a1}} = 1.35$$

$M_{e2} = 1.35 \cdot 30\ Nm = 40.5\ Nm$

The motor torque increases by 35%, resulting in a torque of 40.5 Nm after the change.

TASK 2

For a brushless DC motor, the specification is given:

- Nominal voltage: $U_M = 24\ V$
- Nominal current: $I_M = 4\ A$
- Nominal speed: $v_M = 3000\ rev/min$
- Number of poles: $p = 8$
- Torque constant: $k_t = 0.5\ Nm/A$
- Speed constant: $k_e = 0.0125\ V/(rad/s)$
- Winding resistance: $R_a = 1.2\ \Omega$
- Motor efficiency: $\eta = 85\%$

Calculate:

- Nominal torque: $T_M = ?$
- Nominal angular velocity $\omega_M = ?$
- Induced voltage at rated speed: $U_i = ?$
- Motor input power: $P_{IN} = ?$
- Motor output power: $P_{OUT} = ?$

Solution:

Br A brushless DC motor (BLDC) is a type of DC electric motor where commutation is performed electronically instead of with brushes. This approach enables greater efficiency, longer lifespan, lower maintenance, and better performance compared to traditional brushed motors:

- Higher efficiency: without brushes, there are no friction losses, and thermal losses are reduced.
- Longer lifespan: there are no brushes that wear out over time and need replacement.
- Lower maintenance: there is no need to replace brushes.
- Quieter Operation: there is no noise caused by brushes.

To calculate, we use the equations from the previous task. The constant K_b in the torque equation can also be called the torque constant and denoted as K_t. The rated torque of the motor is the torque at the rated current:

$$M_M = K_t I_M = 0{,}5 \frac{Nm}{A} \cdot 4\ A = 2\ Nm$$

We calculate the rated angular velocity from the given rated speed:

$$\omega_M = v_M \frac{2\pi}{60} = 3000\ \frac{rev}{min} \cdot \frac{2\pi}{60} = 314.16 \frac{rad}{s}$$

The induced voltage can be calculated with the equation:

$$U_i = K_b \omega_M$$

where the constant K_b is also called the voltage constant or electrical constant and denoted as K_e.

$$U_i = K_e \omega_M = 0.0125 \frac{V}{\frac{rad}{s}} \cdot 314.16 \frac{rad}{s} = 3.93\ V$$

When expressed in the appropriate units, the constants K_t and K_e have the same value.

The input power is the power supplied to the motor from the DC source to the rotor. At the rated values, the input power is:

$$P_{IN} = U_M I_M = 24\ V \cdot 4\ A = 96\ W$$

The output power of the motor is the portion of the input power that is converted into mechanical power for rotating the motor. Calculate it with the given efficiency:

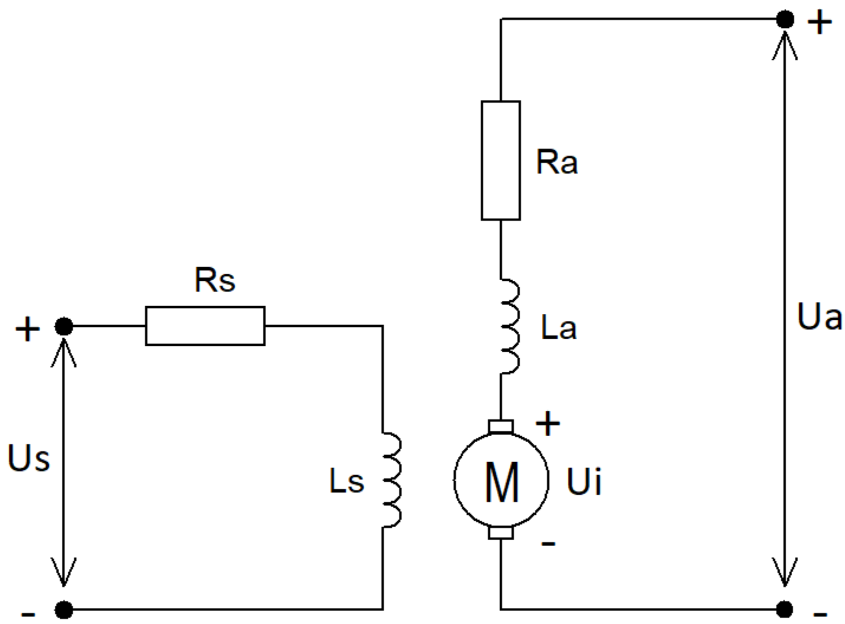
$$P_{OUT} = P_{IN}\, \eta = 81.6\ W$$

In the last equation, we did not consider the input power for powering the stator winding. This can be done if the stator is made with permanent magnets that do not require external power.

TASK 3a

A $220\ V$ $15\ kW$ **separately excited electric motor** has an armature winding resistance of $0.4\ \Omega$. At full load, the electric current on the motor is $70\ A$. Determine the voltage drop on the armature winding and the induced voltage on the armature (back EMF).

Circuit:



Data:

$U_M = 220\ V$

$P_M = 15\ kW$

$R_a = 0.5\ \Omega$

$I_a = 70\ A$

$U_{Ra} = ?$

$U_i = ?$

Solution:

In DC motors with separate excitation, the stator and rotor are electrically isolated as they are powered by two different DC sources. This allows for greater control over the motor's operation. The excitation current flows through the windings on the stator, creating the magnetic field needed for the rotor to turn. The current to the rotor winding comes from a separate source through the commutator and brushes. This setup enables independent control of the current through both windings. By implementing appropriate feedback, it is possible to ensure that the torque is almost constant and independent of the motor's rotational speed.

Main features and advantages:

- Speed control: allows very precise control over speed, as the voltage on the stator can be adjusted independently of the voltage on the rotor.
- High torque at startup and low speeds: separate excitation provides a strong magnetic field, enabling high initial torque.
- Flexibility: due to separate excitation, the motor can be adapted for different applications and required loads.
- Complexity: due to separate power supplies, the construction is more complex, resulting in higher cost and a more demanding maintenance.

Separately excited motors are very popular in industrial applications where robust speed control and high torque are required, such as in elevators, cranes, large industrial fans, and other applications requiring precise speed control under varying loads. This type of motor is also used in railway drives and some types of electric vehicles..

In the task, we have data on the rotor winding current at full load. This directly determines the voltage drop across the rotor winding. Using Ohm's law:

$$U_{Ra} = I_a R_a = 70\,A\ \cdot 0.4\,\Omega = 28\,V$$

In this state, the remaining voltage from the DC source to the rotor, if we neglect other losses, is in the form of induced voltage:

$$U_i = U_a - U_{Ra} = 220\,V - 28\,V = 192\,V$$

TASK 3b

A separately excited motor operates at rated speed and rated torque. Determine the efficiency of the motor at the operating point.

Motor data:

- rated power: $1500\ kW$
- rated voltage: $600\ V$
- rated current: $2650\ A$
- rated speed: $600\ \frac{rev}{min}$
- voltage drop on brushes: $U_B = 2\ V$
- input power to stator winding: $50\ kW$
- impedance of rotor winding: $R_a = 0.003645\ \Omega, L_a = 0{,}1\ mH$
- motor friction torque coefficient: $B_f = 15\frac{Nm}{\frac{rad}{s}}$

The current in the stator winding is constant, while the voltage on the rotor winding can vary.

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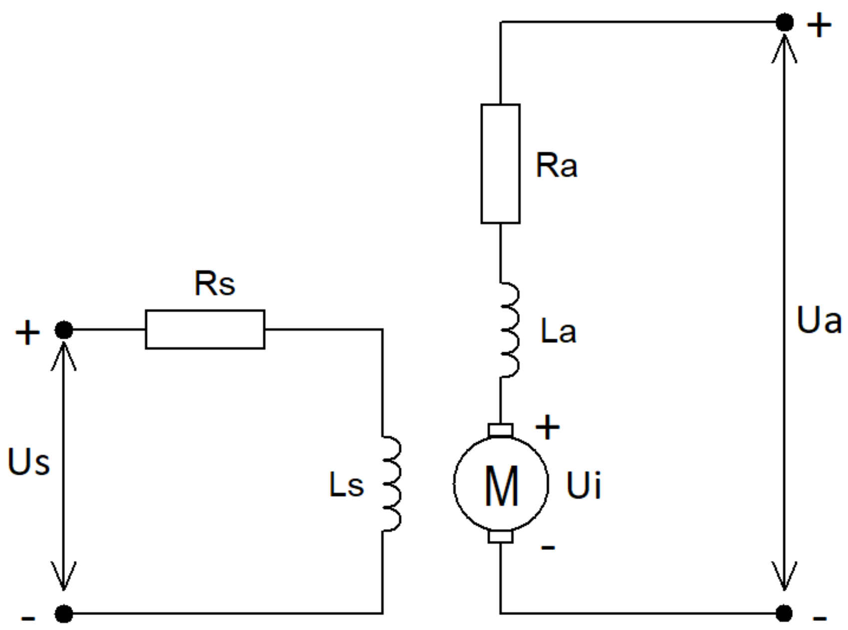


Solution:

To calculate the input power supplied to the motor, we need to determine the voltage set on the rotor winding. We start with the equation for the electrical circuit on the rotor side of the motor, considering the voltage drop on the brushes. In steady state, the following holds:

$$U_a = U_i + R_a I_a + U_B$$

The induced voltage can be expressed using the voltage constant and the angular velocity:

$$U_a = K_e \omega_M + R_a I_a + U_B$$

In this equation ω_M is the rated angular velocity, I_a is the rated current on the rotor, and U_K is the voltage drop on the rotor brushes. The only unknown in the equation is the voltage constant K_e.

In one of the previous tasks, we mentioned that the voltage constant is equal to the torque constant in magnitude:

$$K_e = K_t$$

The torque constant can be calculated from the torque equation:

$$K_t = \frac{M_N}{I_a}$$

M_N is the rated electromagnetic torque produced by the motor, which is the sum of the rated shaft torque M_s (output torque) and the friction torque M_f.

The rated shaft torque can be determined from the motor's output power and the rated angular velocity.

$$\omega_M = v_M \frac{2\pi}{60} = 600 \frac{2\pi}{60} = 62.83 \frac{rad}{s}$$

$$M_s = \frac{P_M}{\omega_M} = \frac{1500\ kW}{62{,}83 \frac{rad}{s}} = 23.873\ kNm$$

The friction torque is calculated from the torque coefficient and angular velocity:

$$M_f = B_f \omega_M = 15 \frac{Nm}{\frac{rad}{s}} 62.83 \frac{rad}{s} = 942.45\ Nm$$

The total electromagnetic torque is then:

$$M_N = M_s + M_f = 23873\ Nm + 942.45\ Nm = 24815.45\ Nm$$

The torque constant is:

$$K_t = \frac{24815.45\ Nm}{2650\ A} = 9.364 \frac{Nm}{A}$$

Returning to the initial equation, we calculate the voltage on the rotor winding:

$$U_a = K_e \omega_M + R_a I_a + U_B = 9.364 \frac{Nm}{A} \cdot 62.83 \frac{rad}{s} + 0.003645\ \Omega \ \cdot 2650\ A + 2\ V \\ = 600\ V$$

The total input power is the sum of the power supplied to the stator winding and the power supplied to the rotor winding:

$$P_{in} = P_{in,f} + P_{in,a} = P_{VHin,f} + U_a\ I_a = 50\ kW + 600\ V \cdot 2650\ A = 1640\ kW$$

Given the motor's output power, we can calculate its efficiency:

$$\eta = \frac{P_M}{P_{in}} = \frac{1500\ kW}{1640\ kW} = 91.46\ \%$$

TASK 4

The **DC electric motor with parallel excitation (i.e. shunt motor)** operates at $250\ V$, its rated power is $10\ kW$. The resistance of the armature (rotor) winding is $0.5\ \Omega$ and the resistance of the stator winding is $200\ \Omega$. Calculate:

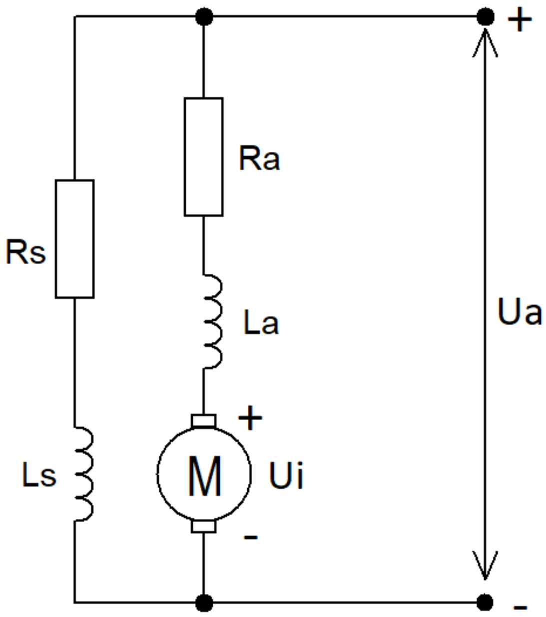
a) current on the rotor winding,

b) current on the stator winding,

c) total current flowing from the supply,

d) efficiency if the total mechanical losses are equal to 500 W.

Circuit:



Data:

$U_M = 250\ V$

$P_M = 10\ kW$

$R_a = 0.5\ \Omega$

$R_s = 200\ \Omega$

$P_{LOSS} = 500\ W$

$I_a, I_s, I = ?$

$\eta = ?$

Solution:

A DC motor with parallel excitation, also known as a shunt motor, is a type of DC motor where the field windings are connected in parallel (or in a shunt) with the armature windings. This configuration creates a stable operating characteristic and allows for better speed control under varying loads.

Characteristics of a DC Shunt Motor:

- Constant magnetic field: The stator windings are connected to a DC source that generally does not change during operation. Therefore, the magnetic field of the stator (magnetic flux ΦΦ) remains relatively constant regardless of the motor's load. This ensures that the motor's speed remains almost constant even when the load changes.
- Speed control: The speed of the motor can be regulated by decreasing or increasing the voltage on the field windings. A DC shunt motor can maintain almost constant speed from no load (idle) to full load.
- High efficiency: Since the magnetic field does not significantly change, these motors are very efficient at constant speeds.

Due to their ability to maintain constant speed and efficient operation under varying loads, DC shunt motors are often used in applications requiring stable speed, such as:

- drives for grinding and polishing machines
- spinning and weaving machines
- fans, blowers
- pump drives, conveyor belts
- other industrial machines requiring stable speed under changing loads

Let's examine the operation of a DC shunt motor in more detail, particularly its self-regulation of speed under different loads. A DC voltage source drives current through two parallel branches, the stator winding, and the rotor winding. The relationship between the currents can be expressed using Kirchhoff's current law:

$$I = I_f + I_a$$

Considering the general equation for the rotor side of a DC motor:

$$U_a = U = U_i + R_a I_a$$

we can write:

$$U = U_i + R_a(I - I_f)$$

If the DC supply voltage does not change, the current in the stator winding remains constant:

$$I_f = \frac{U}{R_s} = const.$$

Given the equation:

$$\Phi_f = \frac{N_f I_f}{\mathcal{R}_m}$$

we can conclude that with a constant stator winding current, the magnetic flux of the stator magnetic field will also be constant. The DC shunt motor is thus called a constant magnetic flux motor.

For the windings of a DC shunt motor, the resistance of the stator winding is significantly higher than the resistance of the rotor winding. The rotor winding must have low resistance to minimize heat losses, thus improving efficiency. Since currents through the rotor winding are relatively high, a low resistance winding is necessary, typically ranging from a few hundredths or tenths to one ohm. Since currents through the stator winding are low, there is no need for very low resistance there. Typical values range from a few tens to a few hundred ohms.

A significant characteristic of the DC shunt motor is its ability to self-regulate its speed when the load changes. For example, if the motor is running without a load and then a load is applied, the speed does not change significantly. Here is what happens when a load is applied:

- Initially, the speed decreases slightly due to the load torque.
- As the motor speed decreases, the induced voltage in the rotor winding also decreases, according to::

$$U_i = K_b \omega_M$$

- Since the DC supply voltage remains unchanged, the voltage across the rotor winding increases as the induced voltage decreases:

$$U = U_i + R_a I_a = U_i + U_{Ra}$$

$$U_{Ra} = U - U_i$$

- Due to the increased voltage across the winding, the current in the rotor winding increases:

$$I_a = \frac{U_{Ra}}{R_a}$$

- The increased current in the rotor winding increases the motor torque, as:

$$M_e = K_b I_a$$

With increased load torque, the motor speed initially decreases slightly, but due to the motor's design and operation, the electromagnetic torque increases, thus increasing the motor speed. Regardless of the load, the speed of a DC shunt motor remains almost constant, hence it's also known as a constant speed motor.

The simplest way to control DC shunt motors is by changing the supply voltage. Another method, without changing the supply voltage which affects both the stator (stator current and magnetic flux) and the rotor (rotor current, motor torque), is by using variable resistors (rheostats) in the stator or rotor windings. This allows independent adjustment of the stator and rotor parameters.

We need to calculate the currents in the motor and its efficiency. Analysing the motor in steady state, where transient effects are negligible and only resistances affect current magnitudes (not inductances), we calculate the stator winding current as::

$$I_f = \frac{U}{R_s} = \frac{250\ V}{200\ \Omega} = 1.25\ A$$

The rotor winding current is determined by the winding resistance and the induced voltage:

$$I_a = \frac{U - U_i}{R_a} = \frac{250\ V - 232\ V}{0.5\ \Omega} = 36\ A$$

The total current from the DC source is:

$$I = I_f + I_a = 37.25\ A$$

To calculate the motor efficiency, we determine all power components in the system:

- Mechanical power of the motor = rated motor power: $P_M = 10\ kW$
- Ohmic losses in the stator: $P_f = I_f^2 R_f = 312.5\ W$
- Ohmic losses in the rotor: $P_a = I_a^2 R_a = 648\ W$
- Mechanical losses of the motor: $P_{LOSS} = 500\ W$

The sum of all power components represents the input power required to maintain this system state:

$$P_{IN} = P_M + P_f + P_a + P_{LOSS} = 11460.5\ W$$

The motor efficiency is therefore:

$$\eta = \frac{P_M}{P_{IN}} = \frac{10000\ W}{11460.5\ W} = 87.26\ \%$$

TASK 4b

A $240\ V$ **DC electric motor with parallel excitation (i.e. shunt motor)** has an armature (rotor) winding resistance of $0.25\ \Omega$, and a stator winding resistance of $120\ \Omega$. At full load, there is a current of $40\ A$ on the armature, and the motor rotates at a speed of $1100\ rpm$. Calculate:

a) The torque developed by the motor.

b) If we change the rheostat on the stator so that the stator winding resistance is $150\ \Omega$, at what speed does the motor rotate if the armature torque and current remain the same?

Data:

$U_M = 240\ V$

$R_a = 0.25\ \Omega$

$R_{s1} = 120\ \Omega$

$R_{s2} = 150\ \Omega$

$I_a = 40\ A$

$v_{M1} = 1100\ rev/min$

$T_M = ?$

$v_{M2} = ?$

Solution:

First, we calculate the angular speed of the motor from its rotational speed:

$$\omega_{M1} = v_{M1}\frac{2\pi}{60} = 1100\frac{rev}{min}\frac{2\pi}{60} = 115.2\frac{rad}{s}$$

Using the data for the armature voltage and current, we determine the induced voltage (steady state):

$$U_i = U - I_a R_a = 240\ V - 40\ A \cdot 0.25\ \Omega = 230\ V$$

The induced voltage determines the output (mechanical) power of the motor:

$$P_M = U_i I_a = 230\ V \cdot 40\ A = 9200\ W$$

At this output power, the motor develops a torque:

$$M_M = \frac{P_M}{\omega_{M1}} = \frac{9200\ W}{115.2 \frac{rad}{s}} = 79.87\ Nm$$

In the second part of the task, we change the resistance in the stator (shunt) winding to 150 Ω. If the motor torque and the armature current remain the same, the induced voltage on the rotor will also remain unchanged. The stator current will change due to the altered resistance, which means the motor's rotational speed must also change.

The initial stator winding current is:

$$I_{f1} = \frac{U_M}{R_{f1}} = \frac{240\ V}{120\ \Omega} = 2\ A$$

After increasing the stator resistance, the stator current changes to:

$$I_{f2} = \frac{U_M}{R_{f2}} = \frac{240\ V}{150\ \Omega} = 1.6\ A$$

Using the equation that relates the induced voltage, magnetic flux, and rotational speed:

$$U_i = K\Phi_f\omega_M$$

The magnetic flux of the stator's magnetic field is proportional to the current in the winding. Therefore, we can write:

$$\frac{\Phi_{f2}}{\Phi_{f1}} = \frac{I_{f2}}{I_{f1}} = \frac{1.6\ A}{2\ A} = 0.8$$

When the stator resistance changes from 120 Ω to 150 Ω, the current in the stator decreases by 20%, and the magnetic flux of the stator also decreases by the same percentage.

Since the induced voltage remains unchanged and the torque remains the same, the new speed can be determined using the ratios and the equation for the induced voltage:

$$\frac{U_{i2}}{U_{i1}} = \frac{K\Phi_{f2}\omega_{M2}}{K\Phi_{f1}\omega_{M1}} = 0.8\ \frac{\omega_{M2}}{\omega_{M1}}$$

$$\omega_{M2} = \frac{\omega_{M1}}{0.8} = \frac{115.2\,\frac{rad}{s}}{0,8} = 144\,\frac{rad}{s}$$

Therefore, after the change, the motor rotates at a speed of:

$$v_{M2} = \frac{\omega_{M2}}{\frac{2\pi}{60}} = 1375\,\frac{rev}{min}$$

TASK 5a

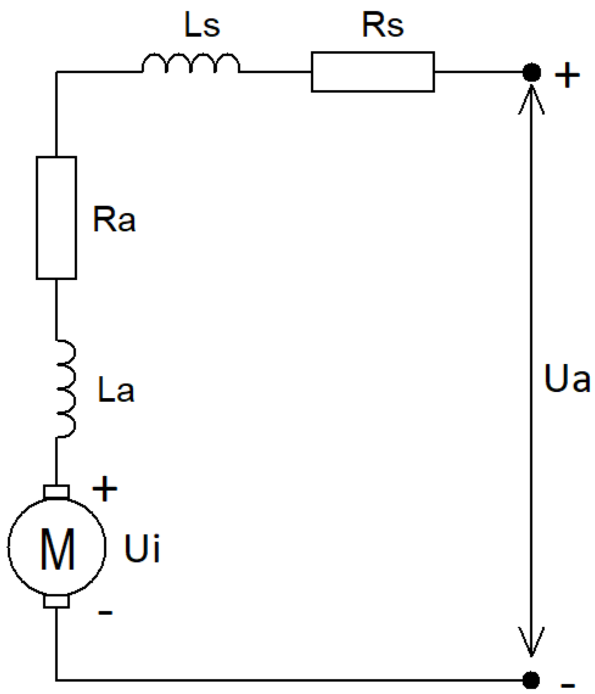
A $230\ V$ **series-excitation brushed electric motor** has a rated speed of $1200\ rpm$ and a current of $37\ A$ through the armature winding. The resistance of the armature winding is $0.4\ \Omega$ and the resistance of the stator winding is $0.2\ \Omega$. The voltage drop across the brushes is $2\ V$. Calculate:

a) rotational speed at a current of $20\ A$ in the armature winding,

b) no-load rotation speed at armature winding current of $1\ A$,

c) rotational speed at 150% of the rated load when the current on the armature winding is $60\ A$ and the magnetic flux of the stator winding is 125% of the flux at the rated load.

Circuit:



Data:

$U_M = 230\ V$

$v_M = 1200\ rev/min$

$I_a = 37\ A$

$R_a = 0.4\ \Omega$

$R_s = 0.2\ \Omega$

$U_B = 2\ V$

$v(I_a = 20\ A) = ?$

$v(no\ load, I_a = 1\ A) = ?$

$v(problem\ c) = ?$

Solution:

A DC motor with series excitation, also known as a series DC motor, is a type of motor that, like the shunt motor, does not require external excitation since both the stator and rotor windings are powered by the same DC source. However, it shares the drawback of being more challenging to independently control the stator and rotor sides of the motor. The key difference lies in the series connection of both windings, meaning the same current flows through both windings. Due to this configuration, the characteristics of the motor differ from those of shunt motors.

The series connection of both windings results in different requirements for the conductors in the windings:

- The current through the circuit (and thus through the stator winding) is relatively higher than in shunt motors.
- Due to the higher current, fewer turns in the stator winding are required to generate the same magnetic force for rotating the rotor winding.
- To minimize losses in the rotor, the resistance of the conductors in the winding must be low, hence the conductors are thicker.

Before we delve into the detailed calculations for series motors, let's review the main characteristics of these motors:

- High starting torque: Series motors develop very high starting torque because the high current through the stator winding results in a strong magnetic field. This enables such motors to start very heavy loads.
- Speed adjusts to load: The motor speed automatically decreases as the load increases, and vice versa. This is due to the dependence of the magnetic field on the current through the stator windings.
- Motor should not be started without load: Because of the high starting torque, an unloaded motor would rotate very quickly, potentially causing damage due to centrifugal forces and mechanical stress.

Due to these characteristics, DC series motors are ideal for applications requiring high starting torque and where the motor speed can vary with the load. They are used in:

- Traction systems: trams, railway vehicles, and electric buses.
- Elevators and cranes: where a strong starting torque is needed for lifting heavy loads.
- Electric tools: where rapid achievement of high power is required.
- Car starters.

Series motors are popular in transport and heavy industrial applications where speed fluctuations are acceptable or even desirable.

For a DC series motor, the current through both windings and the entire system is the same:

$$I = I_f = I_a$$

The basic equation from the equivalent circuit of the motor is:

$$U = U_i + I_f R_f + I_a R_a = U_i + I(R_a + R_s)$$

Due to the equation:

$$\Phi_f = \frac{N_f I_f}{\mathcal{R}_m}$$

the magnetic flux is proportional to the current through the stator winding. Considering the series connection where the currents are equal, the magnetic flux of the stator field is also proportional to the rotor current or the current from the voltage source. Since this current is relatively high, a series motor generates enough magnetic flux to develop sufficient torque for rotation, even with a few turns in the stator winding.

For the series motor, the torque dependence on the current through the stator or rotor winding is:

$$M_e = K_b I_a = \frac{P\,Z}{2\,\pi\,a} \frac{N_f I_f}{\mathcal{R}_m} I_a = \frac{P\,Z}{2\,\pi\,a} \frac{N_f}{\mathcal{R}_m} I_f I_a \propto \, I_f I_a \, \propto \, I_a^2$$

The electromagnetic torque is therefore proportional to the square of the current through the motor. At startup or low speeds, the induced voltage is low, and almost the entire supply voltage is used to create electric current. Considering the low resistance of the windings due to thicker wires, the current through the windings is very high, resulting in high torque. Hence, this type of motor is ideal for applications requiring high starting torque. However, care must be taken not to start the motor without a load, as it could rotate at high speeds uncontrollably.

The torque-speed characteristic of a DC series motor is hyperbolic. At low speeds, the torque is very high, limited only by magnetic flux saturation due to high currents. As the motor speed increases, the induced voltage increases, reducing the voltage across the windings. Consequently, the current through the windings decreases, reducing the torque since torque is proportional to the square of the current.

Speed regulation in series motors is more difficult compared to shunt motors. Series motors do not have the capability to restore speed to the initial value when the load increases. With increased load, the motor speed decreases, reducing the induced voltage. The voltage across the windings is then:

$$U_{coils} = U - U_i$$

Higher voltage across the windings causes higher current:

$$I_f = \frac{U - U_i}{R_a + R_s}$$

The increased current through the stator winding causes saturation of the magnetic core of the stator winding. Consequently, the increase in magnetic flux cannot keep up with the increase in current. Such a magnetic field is not strong enough to provide sufficient magnetic force to restore the motor speed to the value it had before the load increased. Series motors are often used only for short periods to start machines or to move loads with high inertia from a standstill. If such a motor is allowed to run for an extended period, the high currents can destroy the stator winding.

To calculate the motor speed at different current loads, we consider the voltage drop across the armature and brushes and the variable magnetic flux. We also account for changes in the magnetic field due to different currents through the stator windings.

The motor operates at a voltage of 230 V, with the armature winding resistance of 0.4 Ω and stator winding resistance of 0.2 Ω. The voltage drop across the brushes is 2 V. At a rated speed of 1200 rpm and a rated current of 37 A, the induced voltage, which determines the motor speed, is:

$$U_{i,N} = U - I \cdot (R_a + R_s) - U_B = 230\,V - 37\,A\,(0.4\,\Omega + 0.2\,\Omega) - 2\,V = 205.8\,V$$

To calculate the rotational speed at different currents through the rotor winding and under various loads, we use the characteristic equation of a DC motor:

$U_i = K_b \omega_M$

where

$$K_b = \frac{K\ N_f I_f}{\mathcal{R}_m} = \frac{P\ Z}{2\ \pi\ a} \frac{N_f I_f}{\mathcal{R}_m}$$

Since in this task we are interested in the change in speed with varying current in the motor, we combine the two equations and isolate the variable quantities:

$$U_i = \frac{P\ Z}{2\ \pi\ a} \frac{N_f}{\mathcal{R}_m} I_a \omega_M$$

$$\omega_M = \frac{U_i}{I_a} \left(\frac{P\ Z}{2\ \pi\ a} \frac{N_f}{\mathcal{R}_m}\right)^{-1}$$

We see that changing the current through the motor changes the induced voltage and the rotational speed. Let's calculate the rotational speed for three given cases using ratios and starting from nominal values.

a) Rotor current is 20 A

In this case, the induced voltage is:

$$U_{i1} = U - I \cdot (R_a + R_s) - U_B = 230\ V - 20\ A\ (0.4\ \Omega + 0.2\ \Omega) - 2\ V = 216\ V$$

The rotational speed is then:

$$\omega_{M1} = \omega_{MN} \frac{U_{i1}}{U_{iN}} \frac{I_{aN}}{I_{a1}} = 1200 \frac{rev}{min} \frac{216\ V}{205.8\ V} \frac{37\ A}{20\ A} = 2330 \frac{rev}{min}$$

b) Rotor current is 1 A, motor is unloaded

In this case, the induced voltage is:

$$U_{i2} = U - I \cdot (R_a + R_s) - U_B = 230\ V - 1\ A\ (0.4\ \Omega + 0.2\ \Omega) - 2\ V = 227.4\ V$$

The rotational speed is:

$$\omega_{M2} = \omega_{MN} \frac{U_{i2}}{U_{iN}} \frac{I_{aN}}{I_{a2}} = 1200 \frac{rev}{min} \frac{227.4\ V}{205.8\ V} \frac{37\ A}{1\ A} = 49060\ \frac{rev}{min}$$

c) Stator magnetic flux is 125% of nominal values

Since the magnetic flux is increased by 25% relative to nominal values and the flux is proportional to the current through the motor, the current must also increase by 25%. The current is:

$$I_{a1} = 1.25\, I_{aN} = 46.25\, A$$

The induced voltage is then:

$$U_{i3} = U - I \cdot (R_a + R_s) - U_B = 230\, V - 46{,}25\, A\, (0.4\, \Omega + 0.2\, \Omega) - 2\, V = 200.25\, V$$

The rotational speed is:

$$\omega_{M1} = \omega_{MN} \frac{U_{i3}}{U_{iN}} \frac{I_{aN}}{I_{a3}} = 1200 \frac{rev}{min} \frac{200.25\, V}{205.8\, V} \frac{37\, A}{46.25\, A} = 934\, \frac{rev}{min}$$

TASK 5b

A **series-excitation motor**, intended for use in applications requiring variable speed, has the following parameters:

- rated power: $P_M = 3\ HP$
- rated voltage: $U_M = 230\ V$
- rated speed: $v_M = 2000 \frac{rev}{min}$
- impedance of rotor winding: $R_a = 1.5\ \Omega, L_a = 0.12\ H$
- impedance of stator winding: $R_f = 0.7\ \Omega, L_f = 0.03\ H$
- mutual inductance of windings: $M = 0.0675\ H$
- motor friction torque coefficient: $B = 0.0025 \frac{Nm}{\frac{rad}{s}}$

Determine:

a) The supply voltage that ensures the rated torque at the rated speed in steady state .
b) The motor's efficiency at the operating point.

Solution:

From the rated speed, we determine the rated angular velocity:

$$\omega_M = v_M \frac{2\pi}{60} = 2000 \frac{rev}{min} \frac{2\pi}{60} = 209.44 \frac{rad}{s}$$

From the rated power, determine the rated output torque:

$$M_s = \frac{P_M}{\omega_M} = \frac{3\ KM\ \cdot 745.6}{209.44 \frac{rad}{s}} = 10.68\ Nm$$

The friction torque of the motor is:

$$M_f = B\omega_M = 0.0025 \frac{Nm}{\frac{rad}{s}} 209.44 \frac{rad}{s} = 0.52\ Nm$$

The total torque of the motor is thus:

$$M = M_s + M_f = 11.2\ Nm$$

The voltage equation for the series motor in steady state is:

$U = U_i + I_f R_f + I_a R_a$

Expressing the induced voltage with the motor's angular velocity and the current through the stator winding, we get:

$U = M I_f \omega_M + I_f R_f + I_a R_a$

Since the currents are equal, we can simplify the equation:

$U = (M\omega_M + R_f + R_a)\, I_a$

To calculate the required supply voltage, we first need to determine the current through the winding. Considering the equations:

$U_i = K_b \omega_M$

$K_b = M I_f$

$M_e = K_b I_a = M I_f I_a = M I_a^2$

Knowing the total electromagnetic torque of the motor and the mutual inductance of the windings, we can calculate the motor current:

$$I_a = \sqrt{\frac{M_e}{M}} = \sqrt{\frac{11.2\ Nm}{0.0675\ H}} = 12.88\ A$$

The input supply voltage for the motor is thus:

$$U = \left(M\omega_M + R_f + R_a\right)I_a = \left(0.0675\ H \cdot 209.44\frac{rad}{s} + 1.5\ \Omega + 0.7\ \Omega\right)12.88\ A$$
$$= 210.47\ V$$

Now that we know the input voltage and the motor current, we can determine the input (supplied) power:

$P_{VH} = U I_a = 210.47\ V \cdot 12.88\ A = 2710.94\ W$

Since the motor's output power is 3 HP, the motor's efficiency is:

$$\eta = \frac{P_M}{P_{VH}} = \frac{3\ HP\ \cdot 745.6}{2710.94\ W} = \frac{2236.8\ W}{2710.94\ W} = 82.5\ \%$$

TASK 5c

A $100\ V$ series-excited electric motor has an armature winding resistance of $0.2\ \Omega$ and a stator winding resistance of $0.25\ \Omega$. When the motor has a torque of $27.58\ Nm$, it rotates at a speed of $600\ rpm$. The total losses in the core of the windings and losses due to friction are $300\ W$. Determine:

a) lost torque,

b) losses in copper,

c) motor efficiency.

Data:
$U_M = 100\ V$
$T_M = 27.58\ Nm$
$v_M = 600\ rev/min$
$R_a = 0.2\ \Omega$
$R_s = 0.25\ \Omega$
$P_{LOSS} = 300\ W$

$M_{LOSS} = ?$
$P_{Cu} = ?$
$\eta = ?$

Solution:

First, we determine the angular speed of the motor:

$$\omega_M = v_M \frac{2\pi}{60} = \ 3000 \frac{rev}{min} \frac{2\pi}{60} = 314.15 \frac{rad}{s}$$

The given output torque of the motor is used to calculate the motor's output power:

$$P_M = M_M \omega_M = 98\ \mathrm{Nm} \cdot 314.15 \frac{rad}{s} = 30788\ W$$

The lost torque represents the torque that cannot be achieved due to core losses and friction losses. We calculate the lost torque using the ratio of the output power and the output torque of the motor:

$$M_{LOSS} = M_M \frac{P_{LOSS}}{P_M} = 98\ Nm \frac{300\ W}{30788\ W} = 0.95\ Nm$$

The next step is to calculate the current through the motor. We start from the circuit equation, considering that the induced voltage is determined by the product of the voltage constant and the motor's angular speed:

$$U_i = K_e \omega_M = 15.71\ V$$

The motor current is then:

$$I_a = \frac{U - U_i}{R_s + R_a} = \frac{84.29\ V}{1.45\ \Omega} = 58.13\ A$$

The copper losses are therefore:

$$P_{Cu} = I_a^2 (R_s + R_a) = 4900\ W$$

Combining all power components together, we obtain the total or input power of the motor:

$$P_{IN} = P_M + P_{Cu} + P_{IZG} = 35987\ W$$

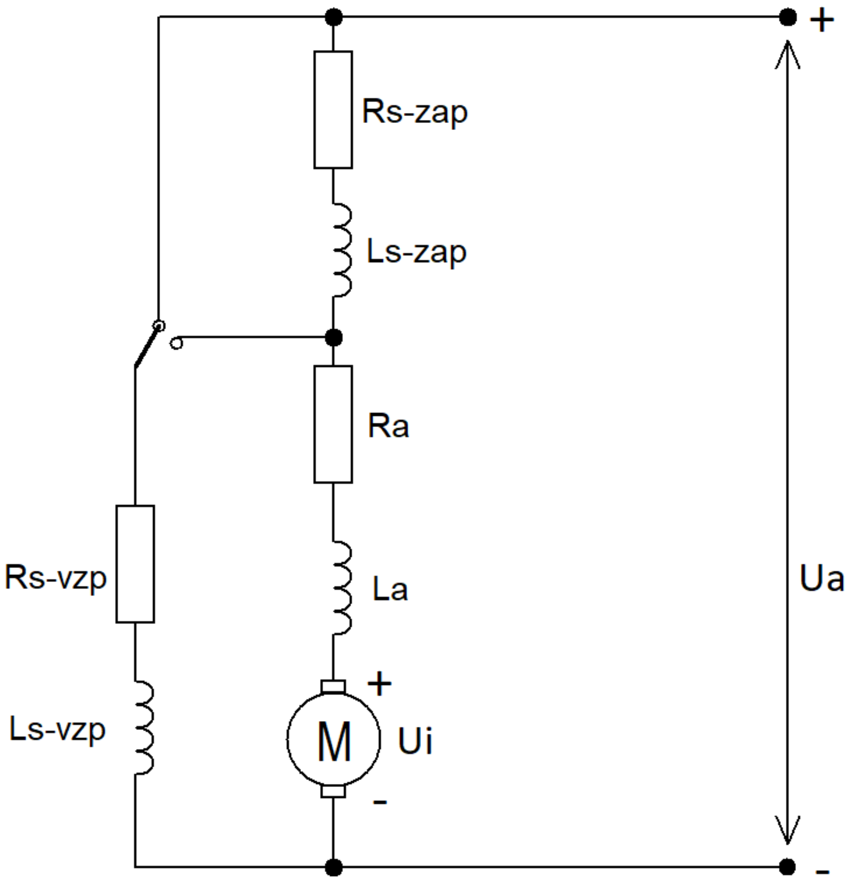
The motor efficiency is thus:

$$\eta = \frac{P_M}{P_{IN}} = \frac{30788\ \ W}{35987\ W} = 85.5\ \%$$

TASK 6

The **electric motor with mixed excitation** has a nominal voltage of $180\ V$ and a nominal power of $6\ kW$. The resistance of the armature winding is $0.3\ \Omega$, the resistance of the winding in the series branch is $0.05\ \Omega$, and the resistance of the winding in the parallel branch is $250\ \Omega$. Determine the rated current and rated speed of the motor if the torque on the motor is $50\ Nm$.

Circuit:



Data:

$U_M = 180\ V$

$P_M = 6\ kW$

$R_a = 0.3\ \Omega$

$R_{s-ser} = 0.05\ \Omega$

$R_{s-par} = 250\ \Omega$

$M_M = 50\ Nm$

$I_M = ?$

$v_M = ?$

Solution:

A DC motor with compound excitation (DC compound motor) is a combination of a shunt motor and a series motor in terms of the connection of the stator and rotor windings. The stator winding consists of two parts: one is connected in series with the rotor winding, and the other is connected in parallel with the rotor winding. There are two possibilities:

- The parallel part of the stator winding can be connected in parallel only with the rotor winding.
- The parallel part of the stator winding can be connected in parallel with the series combination of the rotor winding and the other part of the stator winding.

Another possible division of compound motors is based on the connection between the series stator winding and the parallel stator winding:

- Cumulative compound motors: The magnetic field of the series stator winding is in the same direction as the magnetic field of the parallel stator winding. Such a motor has high torque due to the series winding, while the parallel winding prevents excessive speeds if the load is reduced or removed.
- Differential compound motors: The magnetic field of the series stator winding is in the opposite direction to the magnetic field of the parallel stator winding. When the load increases, the magnetic flux decreases, which ensures a constant or even increasing speed. However, the torque does not increase proportionally with the load. These motors are less commonly used because the decreasing magnetic flux with increasing load can lead to unstable speeds or even "runaway" if the motor is not properly designed.

The combined winding configuration brings the advantages of both shunt and series motors. The main characteristics of DC compound motors are:

- Speed stability: Like shunt motors, these motors maintain speed quite well under different loads but can still adjust speed to some extent if required by a changing load.
- High starting torque: Due to the series part of the stator winding, these motors can develop high starting torque, useful for applications requiring high torque under load.
- Operational flexibility: The ability to adjust the ratio between the parallel and series windings allows for customization to specific application requirements.

DC compound motors are interesting for applications that require a combination of stable speed and high starting torque, such as:

- Lifting and transportation devices: Where lifting heavy loads is necessary, such as elevators and cranes.
- Railway vehicles: Where strong starting torque and stable speed maintenance are needed.
- Industrial machines: Where load fluctuations are common, but stability is also required, such as in presses, metal cutters, and conveyor belts.

The nominal current of the motor is determined from the nominal power and nominal voltage:

$$I_M = \frac{P_M}{U_M} = \frac{6\ kW}{180\ V} = 33.33\ A$$

Since the torque of the motor is known, the angular speed can be determined from the torque equation:

$$\omega_M = \frac{P_M}{M_M} = \frac{6\ kW}{50\ Nm} = 120\frac{rad}{s}$$

We then convert the angular speed to rotational speed:

$$v_M = \frac{\omega_M}{\frac{2\pi}{60}} = 1146\frac{rev}{min}$$