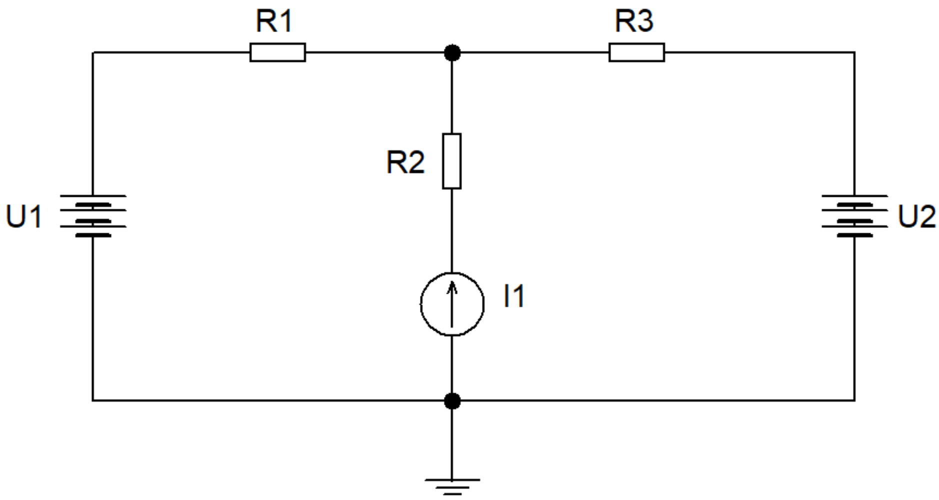
TASK 1

Determine the voltage across resistor R_1. Use the **superposition theorem**.

Circuit:



Data:

$U_1 = 10\ V, \qquad U_2 = 5\ V$

$I_1 = 3\ mA$

$R_1 = 1\ k\Omega, \qquad R_2 = 0.5\ k\Omega, \qquad R_3 = 3\ k\Omega$

$U_{R1} = ?$

Solution:

We are looking for the voltage on resistor R_1; we have three sources in the circuit, two voltage sources and one current source. In the Electrical Engineering and Electronics course in the first year, we usually started solving such tasks by taking a good look at the circuit and trying to find ways to simplify the circuit. We were looking for series and parallel connections, especially resistors. By analysing the above circuit, we find that the resistors are not connected to each other neither in series nor in parallel, but the resistor R_1 is connected in series with the voltage source U_1, the resistor R_2 j is connected in series with the current source I_1, and the resistor R_3 pa is connected in series with the voltage source U_2. Finally, we find out that we have three branches connected in parallel in the circuit.

Since there are three sources in the circuit, and we are interested in the voltage across the resistor R_1, we will use **the superposition theorem** to solve it. Application of the theorem is possible for linear problems, e.g. for calculating voltage or current, but it should not be used for non-linear problems, such as e.g. power calculation. Using the superposition theorem means that we will separately calculate the voltage across the resistor R_1 as a contribution from each of the three sources, and finally add the three contributions together. In general, the contributions of individual sources are added together, but we must pay attention to the polarity of the voltage, which is determined by the polarity of the voltage sources and the direction of the flow of the current sources.

First, we determine the influence of source U_1. The other two sources are removed from the circuit, which means that the voltage source U_2 is replaced with a short circuit, and the current source I_1 is replaced with open terminals. Voltage source U_1, resistor R_1 and resistor R_3 remain in the circuit. These three elements are connected in sequence and they form a loop. Calculating the voltage across one of the resistors means using the voltage divider principle, which is derived from Kirchhoff's voltage law (KVL).

$$\sum_{i=1}^{n} U_i = 0$$

$$U_1 = U_{R1} + U_{R3}$$

The voltage across resistor R_1 is thus:

$$U'_{R1} = U_1 \frac{R_1}{R_1 + R_3} = 2{,}5\ V$$

When determining the influence of the U_2 source, the procedure is similar. We replace the voltage source U_1 with a short circuit, and the current source I_1 with open terminals. Again, we get a loop where the source U_2 and resistors R_1 and R_3 are connected in series. The voltage across resistor R_1 in this case is:

$$U''_{R1} = U_2 \frac{R_1}{R_1 + R_3} = 1{,}25\ V$$

To determine the influence of source I_1, both voltage sources are replaced by a short circuit. The simplified circuit consists of three parallel branches, the branch with the current source I_1, the branch with the resistor R_1 and the branch with the resistor R_3. Since we are interested in the voltage across resistor R_1, we first determine the current across this resistor. The current in the branch with the source I_1 is directly determined by this source, the resistance R_2 does not affect the current. The current I_1 is divided in the node into the current in the circuit with R_1 and the current in the branch with R_3. It is therefore a current divider derived from Kirchhoff's current law (KCL), which applies to nodes. Let's calculate the current in the branch with resistor R_1:

$$I_{R1}''' = I_1 \frac{R_{NAD}}{R_1} = I_1 \frac{R_1||R_3}{R_1} = I_1 \frac{\frac{R_1 R_3}{R_1 + R_3}}{R_1} = I_1 \frac{R_3}{R_1 + R_3} = 2{,}25\ mA$$

The voltage across resistor R_1 is thus:

$$U_{R1}''' = I_{R1}''' R_1 = 2{,}25\ V$$

We calculated the voltages on resistor R_1 as contributions from individual sources. At the end, we have to assemble (add up) the calculated voltages into the total voltage. According to the placement of the source U_1 , the current due to this source will flow from the + terminal up and to the right across the resistor R_1. Given the placement of source U_2 the current due to this source will flow from its + terminal up, to the left across R_3, and part of that current will then flow further left across resistor R_1. Depending on the placement of the source I_1, the current will flow upwards, and then in the node part of this current will flow to the left across the resistor R_1. The contributions of sources I_1 and U_2 are thus added, and the contribution of source U_1 pa is just the opposite, i.e. it is subtracted.

The total voltage across resistor R_1 is thus:

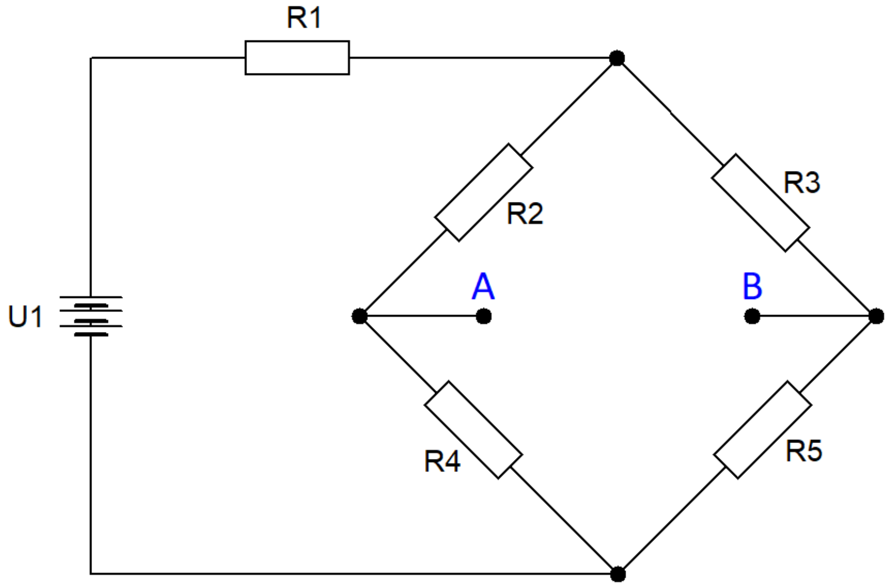
$$U_{R1} = -U_{R1}' + U_{R1}'' + U_{R1}''' = -2{,}5\ V + \ 1{,}25 + \ 2,25 = \ 1\ V$$

TASK 2

If we place a resistor $R_6 = 10\ k\Omega$ between nodes A and B in the Wheatstone bridge in the picture, what current flows through this resistor? Use the **loop current method** to calculate.

Use **Matlab** to solve the matrix equation for loop currents.

Circuit:



Data:

$U_1 = 8\ V$

$R_1 = 10\ k\Omega,\qquad R_2 = 4.7\ k\Omega,\qquad R_3 = 8.2\ k\Omega$

$R_4 = 2.2\ k\Omega,\qquad R_5 = 3.9\ k\Omega$

$I_{R6} = ?$

Solution:

The circuit contains the so-called A **Wheatstone bridge** circuit that is useful for a variety of purposes. One such purpose is **resistance measurement**, where the unknown resistance is one of the four resistances in the bridge. The other three resistors are known, and one of them is adjustable. By changing this resistance, we try to achieve balance, which means that no current flows between points A and B. We then calculate the unknown resistance from the values of the three known resistances. We also met the circuit with the Wheatstone bridge in the first year when directing the alternating voltage. If the circuit is supplied with an alternating voltage (sine) and four appropriately oriented diodes are placed in the bridge instead of resistors, a **full-wave rectified** sine signal is obtained between points A and B.

In the example above, we place a resistor R_6 between points A and B and we want to calculate the current that flows through this resistor. We could start again by simplifying the circuit by looking for series and parallel connections, but even here, we would find that there are no resistors connected in series or parallel.

We see from the circuit that there are some special types of binding. Resistors R_2, R_3 and R_6 make up the so-called triangular bond. Similarly, for resistors R_4, R_6 and R_5, we find that they are in triangular connection. On the other hand, resistors R_2, R_6 and R_4 in the so-called star bindings. This means that all three resistors are connected in the central node of the star (node A). In a similar way, for resistors R_3, R_6 and R_5, we find that they are in a star connection with the central node at point B. In the theory of electrical circuit analysis, there are methods that, by means of the conversion of a delta connection into a star connection or vice versa, enable the calculation of the required quantities.

Another group of methods that can be used to solve arbitrarily large circuits are the so-called computational methods. These methods enable the writing of matrix equations with a systematic approach and the use of basic laws (Ohm's law, KNZ, KTZ). These equations are solved with computer programs (Matlab, Mathematica ...). These are:

- Method of branch currents
- Method of loop currents
- Method of nodal potentials

Branch currents are actual currents and there are as many of them in a circuit as there are branches in the circuit. There are six branches in our circuit, so we should calculate six currents. The **method of branch currents** is, in principle, always the most computationally demanding.

The **nodal potential method** is a procedure for determining the unknown potentials at the nodes. In a circuit, at least one node is usually known - this is the ground point. Since nodes are the points where branches meet, there are usually fewer of them than branches, so this method is computationally less demanding than the branch current method.

The last method required by the task to calculate the current across resistor R_6 is the **method of loop currents**. Loop currents are virtual currents with which we reduce the system of equations obtained by the method of branch currents. Loop currents "flow" in loops, where only closed loops are considered. These loops contain no cross connections (branches). In our circuit, we have three loops:

- loop U1 – $R_1 - R_2 - R_4$
- loop $R_2 - R_6 - R_3$
- loop $R_4 - R_6 - R_5$

For all closed loops, we write the KVL equations, where the voltages on the resistors are expressed by the loop currents I_1, I_2 and I_3. Loop currents are always directed clockwise, regardless of the actual currents in the circuit. In the case of resistors located between two loops, we take into account the influence of both loop currents, which are always opposite due to the direction of the currents (clockwise).

$$I\text{:} \ -U_1 + I_1R_1 + I_1R_2 - I_2R_2 + I_1R_4 - I_3R_4 = 0$$

$$II\text{:} \ I_2R_2 - I_1R_2 + I_2R_3 + I_2R_6 - I_3R_6 = 0$$

$$II\text{:} \ I_3R_4 - I_1R_4 + I_3R_6 - I_2R_6 + I_3R_5 = 0$$

If we arrange the above three equations and write them in matrix form, we get:

$$\begin{bmatrix} R_1 + R_2 + R_4 & -R_2 & -R_4 \\ -R_2 & R_2 + R_3 + R_6 & -R_6 \\ -R_4 & -R_6 & R_4 + R_5 + R_6 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} U_1 \\ 0 \\ 0 \end{bmatrix}$$

The above matrix equation could also be written directly from the circuit without the closed-loop KNZ equations. Each row of the matrix equation represents one loop. The matrix equation consists of a square matrix containing the resistances of the resistors in each loop, a vector of unknown loop currents, and a vector of voltage sources. A square matrix has on its diagonal the sum of the resistances of the resistors in each loop. This means that in loop 1, due to the loop current I_1, we get voltages on resistors R_1, R_2 and R_4. In loop 2 we get voltages on resistors R_2, R_3 in R_6 due to loop current I_2, and in loop 3 we get voltages on resistors R_4, R_6 and R_5 due to loop current I_3. At off-diagonal locations in the square matrix are the resistances of the resistors that lie between the two loops. All off-diagonal elements have a negative sign, since the direction of the loop current in the adjacent loop is opposite to the direction of the "main" current in the loop. When writing the voltage vector on the right side of the matrix equation, we take into account the voltage sources that are in the individual loops. In our case, this is the voltage source U_1, which is in loop 1.

Insert the values into the matrix equation and get:

$$\begin{bmatrix} 16900 & -4700 & -2200 \\ -4700 & 22900 & -10000 \\ -2200 & -10000 & 16100 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \\ 0 \end{bmatrix}$$

We get a system of three linear equations with three unknowns that should be solved. Since it is a relatively small system, it could be solved manually, e.g. by the method of variable elimination or some other approach. The task requires us to use the Matlab tool to solve it. Since it is a computational tool, we can solve a system of any size in this way, even such a large system that would be very time-consuming to solve manually.

In Matlab, we will use the Command window, where we will define all input data (square matrix, voltage vector), and then we will use Matlab's built-in functions to solve the system of linear equations. Let's define a square matrix and a voltage vector::

Command Window

```
>> U=[8;0;0]

U =

     8
     0
     0

>> R=[16900 -4700 -2200; -4700 22900 -10000; -2200 -10000 16900]

R =

        16900       -4700       -2200
        -4700       22900      -10000
        -2200      -10000       16900
```

In the Matlab tool, we define a matrix by writing the values inside square brackets. The values in one row of the matrix are written separated by spaces, and we move to a new row with a semicolon. A voltage vector can be thought of as a vertical (column) vector, so all values in the vector are separated by semicolons.

To solve the system of linear equations, we use the function `linsolve(R, U)`, where the first parameter is the square matrix of resistance, and the second parameter is the column vector of voltage: $\frac{a}{b}$

```
>> I=linsolve(R,U)

I =

   1.0e-03 *

    0.5520
    0.1951
    0.1873
```

We get a solution in the form of a column vector of loop currents. The values of the loop currents are:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0.555\,mA \\ 0.202\,mA \\ 0.2014\,mA \end{bmatrix}$$

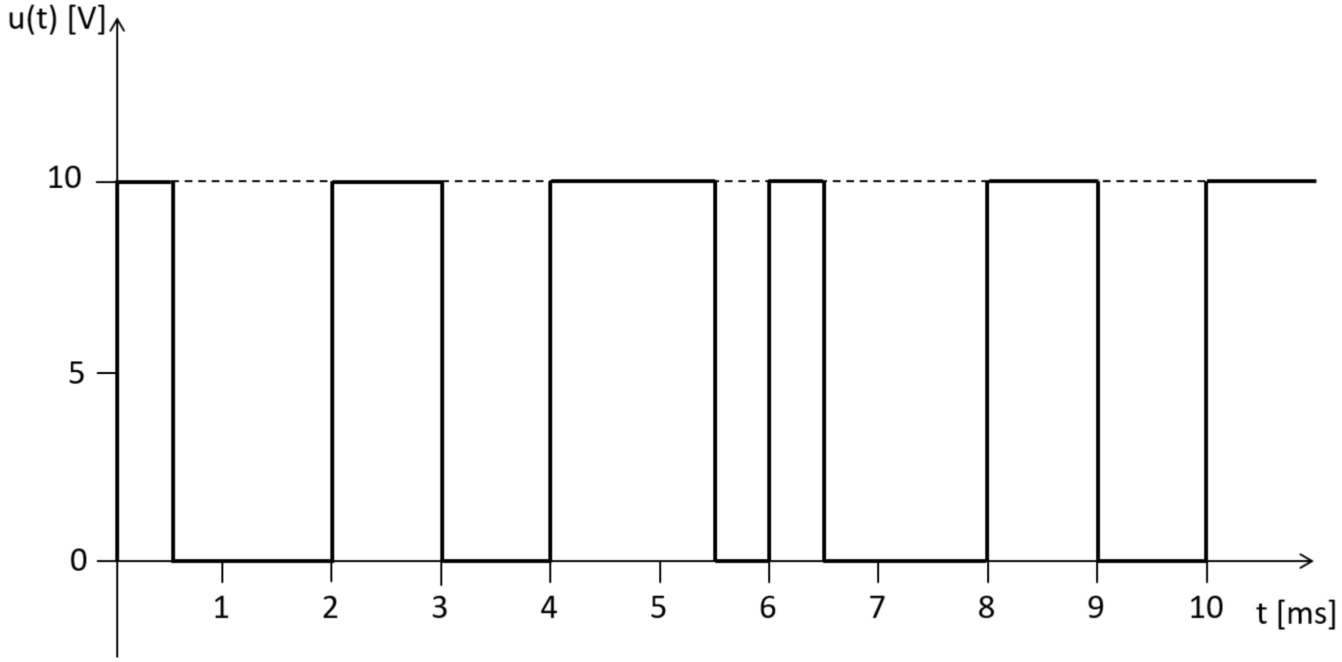
Now that we know the values of the loop currents, we can determine the value of the current across the resistor R_6. Current I_{R6} is a branch current or the current in the branch that lies between loops 2 and 3. The direction of the current is determined from the values of the currents I_2 and I_3. The larger of the two currents determines the direction of the I_{R6} current. The value of the current I_{R6} is obtained by the equation:

$$I_{R6} = I_2 - I_3 = 0.6\ \mu A$$

TASK 3

To control the electric motor, we use the pulse-width modulated signal (PWM) in the picture. What is the average value of the signal and what is its average power if it is observed on a resistor $R = 1\ k\Omega$?

Graph:



$U_{AVG}, P_{AVG} = ?$

Solution:

The goal of the task is to calculate the average value of the signal and its average power. These are two of the characteristic parameters of AC signals used in electrical engineering and electronics for power supply and information transmission.

The image shows a pulse width modulated (PWM) signal. This is a periodic signal that only has two values: the 0 V value (OFF state) or the maximum value (ON state). The maximum value is determined by the system and cannot be changed. For Arduino microcontrollers, this is e.g. voltage 3.3 V or 5 V (microcontroller supply voltage). The information that we want to transmit with the PWM signal is written in the pulse width. The pulse width is usually described by the duty cycle parameter. We express this parameter as a percentage and means the fraction of one period of the PWM signal when this value is ON. If the signal is ON (5 V) for half the period and OFF (0 V) for half the period, the duty cycle is 50%. Important parameters and characteristics of the PWM signal are:

- basic period or signal frequency (time for 100% duty cycle),
- speed of switching between extreme values,
- useful for controlling motors as inert loads that are not sensitive to fast discrete switching,
- limited signal strength (saving).

The graph shows that the basic period of the PWM signal is two milliseconds. The signal consists of three sections with different duty cycle values, and these sections are then repeated. In the first section, the signal has a maximum value (10 V) of 0.5 milliseconds, so the duty cycle is 25%. On the second section, the signal has a value of 10 V for one millisecond; the duty cycle is thus 50%. On the third section, the signal has a value of 10 V for 1.5 milliseconds, which represents a duty cycle of 75%.

To calculate the average value, we start from the basic definition for mean values of the signal:

$$U_{AVG} = \frac{1}{T}\int_0^T u(t)dt$$

We need to calculate the integral of the signal in the area of one period and then divide the result by the length of the period. Since we have three different sections, we will take a value of 6 ms for T. Let's write the integration equation again and insert the actual values.

$$U_{AVG} = \frac{1}{T}(\int_0^2 u_1(t)dt + \int_2^4 u_2 dt + \int_4^6 u_3 dt)$$

We remember from the last year that for known signal forms we can replace the integration equation with a weighted sum:

$$U_{AVG} = \frac{1}{T}\sum_1^N U_{AVGi}T_i$$

For our case, the actual equation is:

$$U_{AVG} = \frac{1}{T}\sum_1^3 U_{AVGi}T_i$$

With the weighted sum, the average value can be calculated significantly faster:

$$U_{AVG} = \frac{1}{T}\sum_{1}^{3} U_{SRi}T_i = \frac{1}{T}(U_{AVG1}T_1 + U_{AVG2}T_2 + U_{AVG3}T_3)$$
$$= \frac{1}{6}(10 \cdot 0.5 + 10 \cdot 1 + 10 \cdot 1.5) = 5\ V$$

To calculate the average power, we need to use the equation:

$$P = \frac{U_{EF}^2}{R}$$

We defined the effective value of the signal as the value that a constant signal having the same power as the basic signal would have. By definition, the effective value of the signal is calculated using the equation:

$$U_{EF}^2 = \frac{1}{T}\int_{0}^{T} u^2(t)dt$$

The equation is similar to the calculation of the mean value, except that the signal is first squared before integration, and at then result is squared. Even with the effective value, the calculation is much simpler if we know the shape of the signal in all sections. Let's write the equation with a weighted sum:

$$U_{EF}^2 = \frac{1}{T}\sum_{1}^{N} U_{EFi}^2 T_i$$

For our example, the effective value is therefore the same:

$$U_{EF}^2 = \frac{1}{T}\sum_{1}^{3} U_{EFi}^2 T_i = \frac{1}{T}(U_{EF1}^2 T_1 + U_{EF2}^2 T_2 + U_{EF3}^2 T_3)$$
$$= \frac{1}{6}(10^2 \cdot 0.5 + 10^2 \cdot 1 + 10^2 \cdot 1.5) = \frac{300}{6} = 50\ V^2$$

$$U_{EF} = \sqrt{U_{EF}^2} = \sqrt{50} = 7.071\ V$$

The average power of the PWM signal is therefore:

$$P = \frac{U_{EF}^2}{R} = \frac{50}{1000} = 50\ mW$$