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Operational Amplifiers

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Operational amplifier is a linear amplifier with a very high amplification. It is shown schematically in Fig. 1. Operational amplifier has four inputs. Input signals $+U_0$ and $-U_0$ are power supply

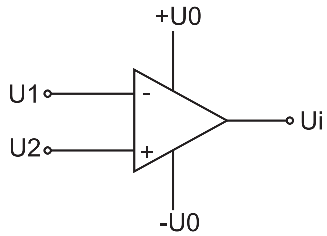

Figure 1: Schematic representation of operational amplifier [5]

signals. Their typical values are $+15V$ and $-15V$ [4]. The relationship between input signals U_1 and U_2 and output signal U_i is given by the following equation

$$U_i = A(U_2 - U_1) \tag{1}$$

A model of the operational amplifier is shown in Fig. 2. The characteristics of the operational

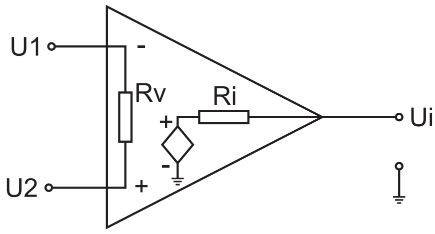

Figure 2: Model of an operational amplifier [2, 6]

amplifier are close to an ideal amplifier, which should have the following properties [5]:

- very high amplification: $A \geq 10^5+$
- very high input resistance: $R_v > 1M\Omega$
- low output resistance: $R_i = 50 - 75\Omega$

In fact the amplification A depends on the input signal(s), time, temperature, supply voltage, etc., and can vary from one to the other amplifier for the factor of 5 [1].

As an example, let's determine the output signals of an operational amplifier for the input signal combinations shown in Fig. 3. In all three cases we will assume that the amplification is

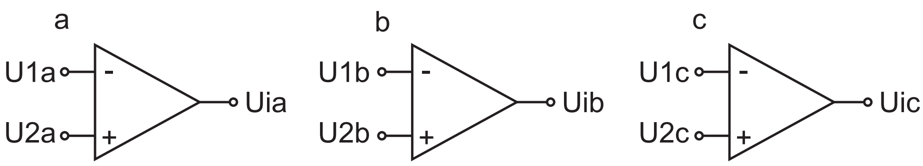

Figure 3: Output signals for specific input signals

equal to $A = 10^5$.

The starting point in all three cases is of course the Eq. 1. In the first case the output voltage U_i is equal to

$$U_i = A\,(U_2 - U_1) = 10^5\,(10\mu V - 10\mu V) = 0V$$

In the second case the output voltage U_i is equal to

$$U_i = A\,(U_2 - U_1) = 10^5\,(5\mu V - 20\mu V) = -1.5V$$

In the third case the output voltage U_i is equal to

$$U_i = A\,(U_2 - U_1) = 10^5\,(25\mu V - 15\mu V) = 1V$$

We can see, that very weak input signals (several $10\mu V$) result in output signals in the range of one volt.

In order to analyze the behaviour of systems which use the operational amplifier as one of the building blocks, the following assumptions are important:

1. **The voltages U_1 and U_2 are equal.**
   We make this assumption, if we take into account that amplification A is very high. In the Ex. **??** we have shown, that a difference of $10\mu V$ results in an output signal (voltage) being equal to $1V$. Because the output signal is limited by the power supply (approximately 15V), the maximal difference of the input signals $|U_2 - U_1| \approx 150\mu V = 0.00015V$. For the vast majority of the practical examples we can therefore assue that voltages U_1 and U_2 are equal.

2. **The input current is equal to 0**
   Because the input resistance R_v is very high (see Fig. 2) and at the same time the difference of the input voltages $U_2 - U_1$ very low, we can assume that the current flowing through resistor R_v is equal to zero. The model of the operational amplifier shown in Fig. 2 can therefore be modified to the form shown in Fig. 4. Because the input current is equal to zero,

   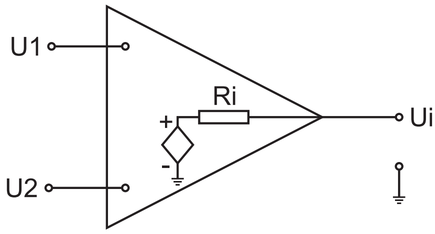

   Figure 4: Simplified model of an operational amplifier

   the input circuit can be opened and the inputs shown separately.

3. **The output resistance is equal to 0**
   This assumption is the most questionable. In fact the output resistance $R_i = 50 - 75\Omega$. The assumption is valid only if the resistance of the load connected to the amplifier is much larger than R_i. In the vast majority of the practical examples this assumption is fulfilled.

With these assumptions we will analyze some of the most common applications of the operational amplifier. The first one will be the so called voltage follower shown in Fig. 5. If we take the Eq. 1

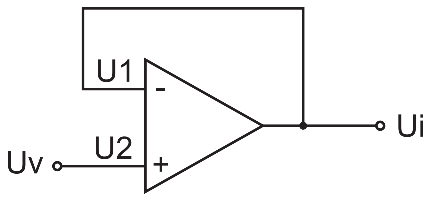

Figure 5: Voltage follower

as a starting point, we can get the following relationship between inputs and the output

$$U_i = A\,(U_2 - U_1) = A\,(U_v - U_1) = A\,U_v - A\,U_i \quad\Rightarrow\quad U_i = \frac{A}{1+A}\,U_v \approx U_v$$

because $A >> 1$.

The voltage follower is usually used to solve the loading problem. An example of the loading problem is shown in Fig. 6. The sensor output is a voltage of $10V$, which should be the input

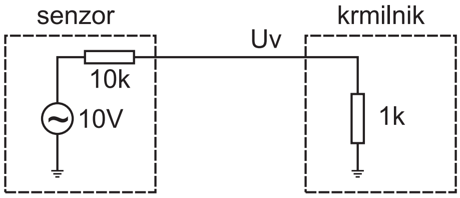

Figure 6: Loading problem

voltage for the controller. Because the output resistance of the sensor is equal to $10k\Omega$ and the input resistance to the controller $1k\Omega$ we get the following voltage as the input to the controller

$$U_v = \frac{1k\Omega}{10k\Omega + 1k\Omega} 10V \approx 0.91V$$

The problem can be solved by putting a voltage follower between sensor and controller as shown in Fig. 7. Because on the input side of the operational amplifier (voltage follower) no current is

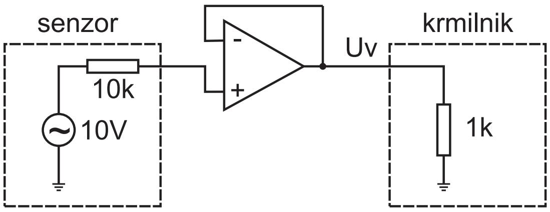

Figure 7: The solution of loading problem using voltage follower

flowing, we have no voltage drop on the resistor in the sensor. The voltage follower input voltage is therefore equal to $10V$. We get the same voltage on the output of the voltage amplifier and therefore we can write $U_v = 10V$.

Inverting amplifier (see Fig. 8) is probably the most commonly used application of the operational amplifier. We only need to resistors to realize it. Resistor R is on the input side and resistor

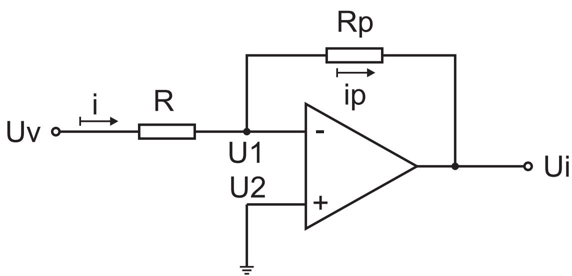

Figure 8: Inverting amplifier

R_p in the feedback. It is called inverting amplifier because the input voltage is connected to inverting input of the operational amplifier. Because non-inverting input is connected to ground, the voltage $U_2 = 0$. Due to the assumption of equal input voltages, we can also write $U_1 = 0$. As we have also assumed that input current to the amplifier is zero, we can write $i = i_p$. This gives us

$$\frac{U_v - 0}{R} = \frac{0 - U_i}{R_p} \quad \Rightarrow \quad U_i = -\frac{R_p}{R} U_v$$

The ratio between output and input voltage determines the amplification A_s of the inverting amplifier. It is therefore given by the following equation.

$$A_s = -\frac{R_p}{R}$$

It should however be emphasized, that amplification A_s cannot be larger than the amplification A of the operational amplifier.

If the input voltage U_v is however connected to the non-inverting input, we get the so called non-inverting amplifier shown in Fig. 9. Because the input current to the operational amplifier is

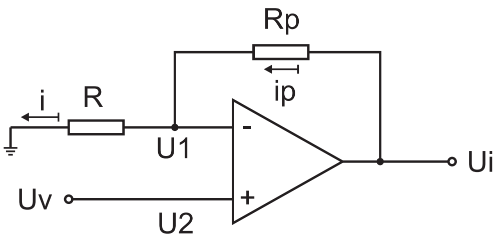

Figure 9: Non-inverting amplifier

zero, we must have $i = i_p$. As the input voltages U_1 and U_2 are equal and additionally $U_v = U_2$, we can write

$$\frac{U_v - 0}{R} = \frac{U_i - U_v}{R_p} \quad \Rightarrow \quad U_i = \left(\frac{R_p}{R} + 1 \right) U_v$$

The amplification of the non-inverting amplifier is therefore given by the following equation.

$$A_s = \frac{R_p}{R} + 1$$

Of course, also in this case it cannot be larger than the amplification A of the operational amplifier.

A more accurate model of the operational amplifier is given by the following equation [3]

$$U_i = \frac{10^7}{D+1}(U_2 - U_1)$$

Let's prove that the amplifier shown in Fig. 10 is unstable. We can write

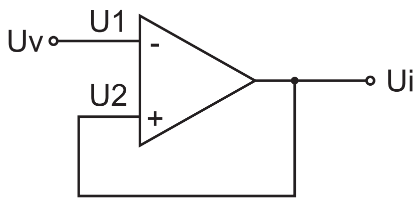

Figure 10: Unstable application of an operational amplifier

$$U_i = \frac{10^7}{D+1}(U_2 - U_1) = \frac{10^7}{D+1}(U_i - U_v)$$

which gives us

$$U_i = \frac{-10^7}{D+1-10^7} U_v \approx \frac{-10^7}{D - 10^7} U_v$$

Because the characteristic equation has a zero with a positive real component (zero in the right half of the complex plane) the system is unstable.

The next important application of the operational amplifier is the so called summing amplifier, shown in Fig. 11. In a similar way as in the case of the non-inverting amplifier we can conclude that $U_1 = U_2 = 0$. We must also have $i_a + i_b + i_c = i_p$. Consequently we can write

$$\frac{U_a - 0}{R_a} + \frac{U_b - 0}{R_b} + \frac{U_c - 0}{R_c} = \frac{0 - U_i}{R_p} \quad \Rightarrow \quad U_i = -\left(\frac{R_p}{R_a} U_a + \frac{R_p}{R_b} U_b + \frac{R_p}{R_c} U_c \right)$$

If the resistances R_a, R_b in R_c are equal, we can write $R_a = R_b = R_c = R$. In this case we get the following equation

$$U_i = -\left(\frac{R_p}{R} U_a + \frac{R_p}{R} U_b + \frac{R_p}{R} U_c \right)$$

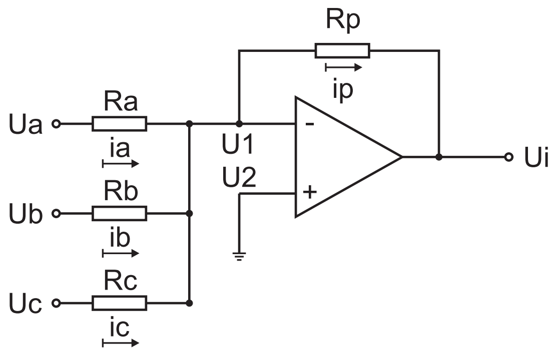

Figure 11: Summing amplifier

The functioning of summing amplifier was shown for three input signals. The same logic applies if there are only two input signals, or if there are more than three input signals.

A very interesting application of operational amplifier is the so called differential amplifier, shown in Fig 12. It is similar to the inverting amplifier. The only difference is that in this case the

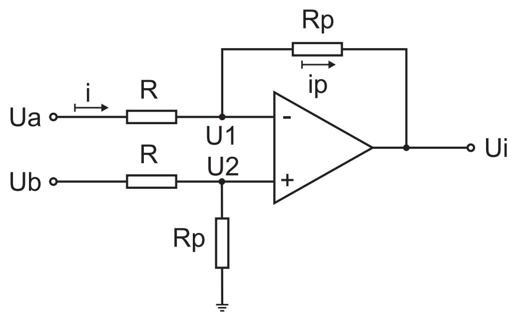

Figure 12: Differential amplifier

non-inverting input is not grounded. The voltage U_2 can in this case be written as $U_2 = \frac{R_p}{R+R_p} U_b$. Due to the assumption of equal input voltages we can write $U_1 = U_2$. Using the assumption that no current flows through the operational amplifier, we can write

$$i = i_p \quad \Rightarrow \quad \frac{U_a - U_1}{R} = \frac{U_1 - U_i}{R_p} \quad \Rightarrow \quad \frac{U_a - \frac{R_p}{R+R_p} U_b}{R} = \frac{\frac{R_p}{R+R_p} U_b - U_i}{R_p}$$

After some calculation we get

$$U_i = \frac{R_p}{R}(U_b - U_a)$$

Differential amplifier is the staring point for the so called instrumentation amplifiershown in Fig. 13. Instrumentation amplifier is a differential amplifier with two voltage followers on the input side.

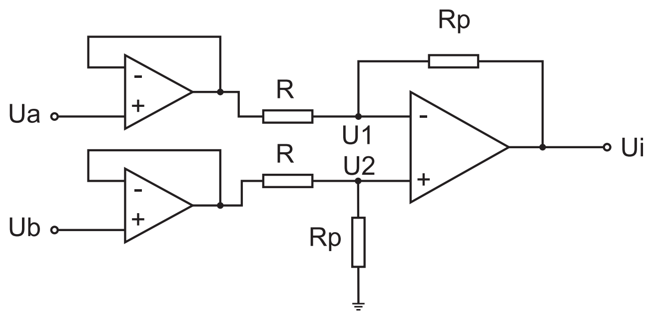

Figure 13: Instrumentation amplifier

The voltage followers are added to get the following properties [5]:

- The input resistance is increased to such extent that we don't have the loading problem anymore.
- Both input resistances are the same.
- Resistors R and R_p are separated from the input signals U_a and U_b. Consequently instrumental amplifiers can be made by predetermined amplification.

There are also instrumentation amplifiers with programmable amplification [5]. They are for example used in AD converters where the same converter is used to digitize signals of very different voltage ranges.

For control purposes integrators and differentiators are of extreme importance. Integrator is shown in Fig. 14. The analysis of the integrator is similar as in the case of inverting amplifier. The

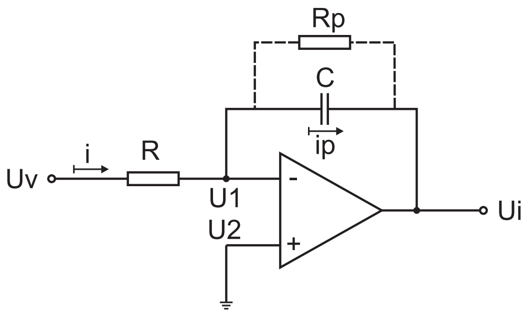

Figure 14: Integrator

starting point is again the equality of voltages U_1 and U_2 and assumption that no current flows through the input. We get the following result.

$$i = i_p \quad \Rightarrow \quad \frac{U_v - 0}{R} = \frac{0 - U_i}{\frac{1}{CD}} \quad \Rightarrow \quad U_i = -\frac{1}{RCD} U_v$$

The transfer characteristic $-\frac{1}{RCD}$ represents an integral nature. Because of the problem of saturation an additional resistor R_p (denoted with a dashed line in Fig. 14 is added. If the resistance of the resistor R_p is at least ten times bigger than the resistance of the resistor R the performance of the integrator doesn't deteriorate significantly [5].

Diferentiator is shown in Fig. 15. We use the same assumptions as in the case of the integrator.

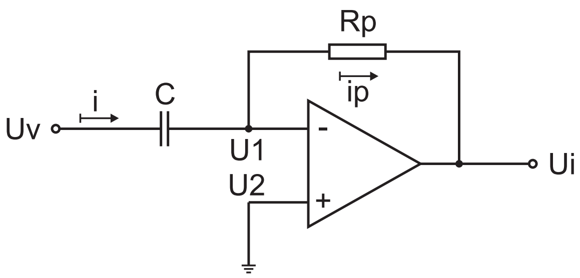

Figure 15: Diferentiator

Here we get the following result.

$$i = i_p \quad \Rightarrow \quad \frac{U_v - 0}{\frac{1}{CD}} = \frac{0 - U_i}{R} \quad \Rightarrow \quad U_i = -RCD\, U_v$$

The transfer characteristic $-RCD$ is of course of a derivative type.

We can make a PID controller using the circuits presented above. It is composed of three components (proportional P, integral I and derivative D), as shown in Fig. 16. Its transfer charac-

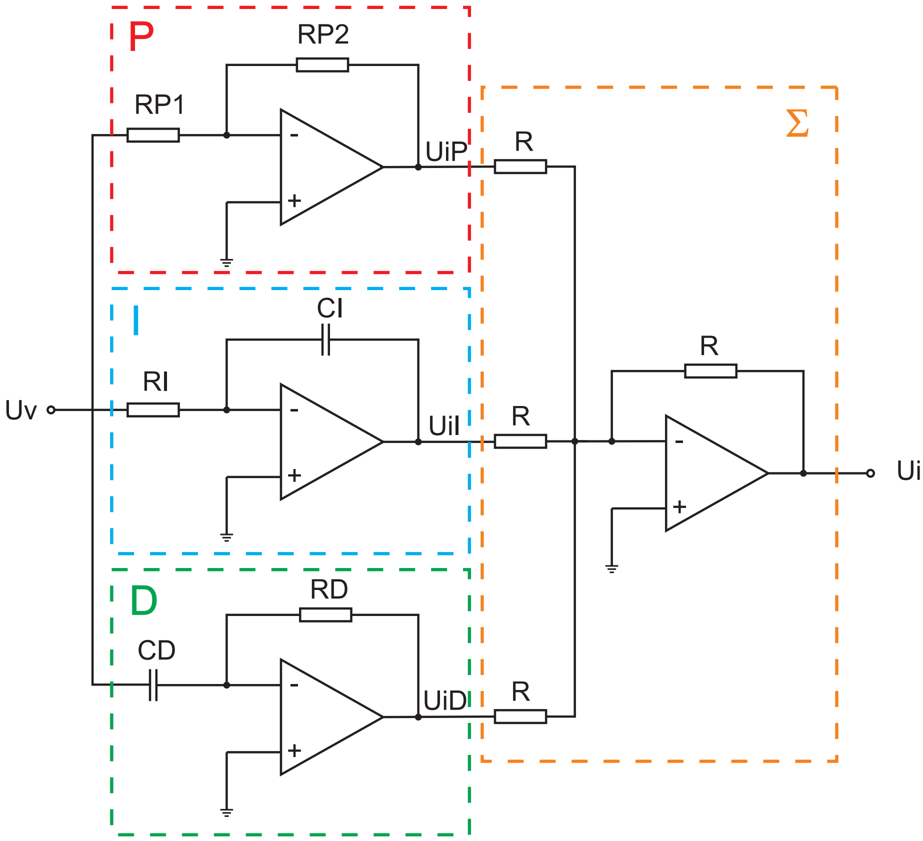

Figure 16: PID controller

teristics is equal to

$$\frac{U_i(t)}{U_v(t)} = P(D) = K_P + \frac{K_I}{D} + K_D D$$

The proportional component is given by

$$\frac{U_{iP}(t)}{U_v(t)} = -\frac{R_{P2}}{R_{P1}}$$

The second component is integral, defined with

$$\frac{U_{iI}(t)}{U_v(t)} = -\frac{1}{R_I C_I D}$$

and finally derivative

$$\frac{U_{iD}(t)}{U_v(t)} = -R_D C_D D$$

The outputs of these three components U_{iP}, U_{iI} in U_{iD} need to be added. This can be done by the summing amplifier. The value of R is not important. It's only important that all three resistors have the same resistance R. As summing amplifier is also inverting the signal, it cancels the negative sign obtained in outputs of all three components. The output voltage is therefore equal to

$$U_i(t) = -U_{iP}(t) - U_{iI}(t) - U_{iD}(t) = \left(\frac{R_{P2}}{R_{P1}} + \frac{1}{R_I C_I D} + R_D C_D D\right) U_v(t)$$

Fig. 16 is showing PID controller with an ideal D component. Real one can be obtained by a addition of another resistor R_m in fron of capacitor C_D. In this case the output voltage $U_i(t)$ is equal to

$$U_i(t) = -U_{iP}(t) - U_{iI}(t) - U_{iD}(t) = \left(\frac{R_{P2}}{R_{P1}} + \frac{1}{R_I C_I D} + \frac{R_p C_D D}{R_m C_D D + 1}\right) U_v(t) \quad (2)$$

We the circuit known we just have to select the right values of capacitances and resistances to get the desired transfer characteristics. In order to make it easier for implementation a schematic of PID controller realized by LM741 operational amplifier is shown in Fig. 17).

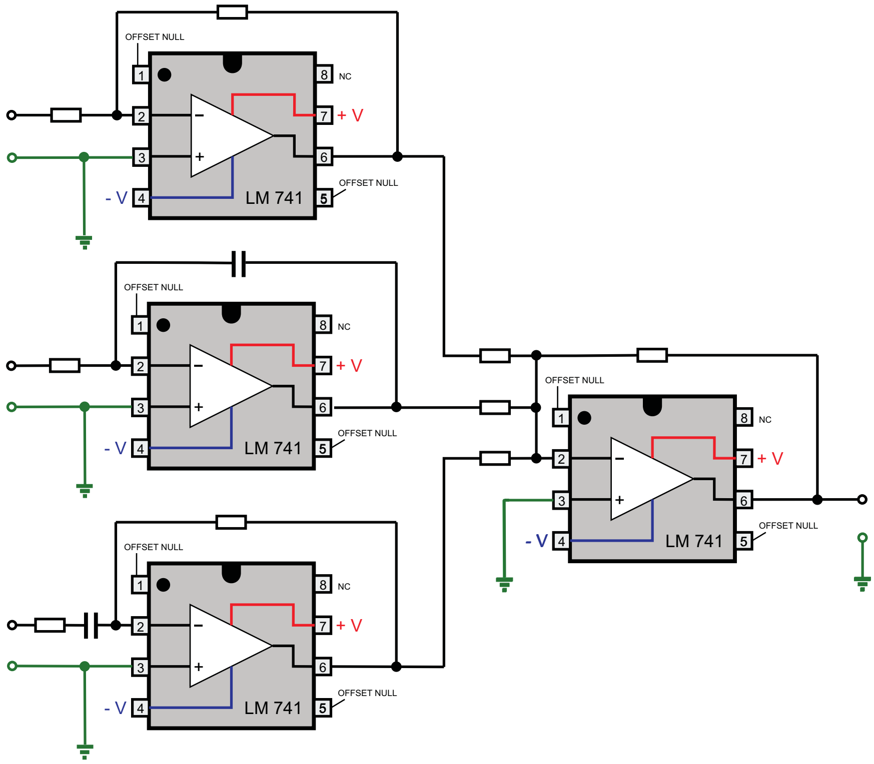

Figure 17: Four LM741 operational amplifiers resulting in a PID controller

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