# System Dynamics modelling

**Paweł Litwin**
**Rzeszów University of Technology**

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# Guide to Stocks and Flows Diagrams

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# Introduction to Stocks and Flows Diagrams (SFD)

- **Definition of SFD:** Stocks and Flows Diagrams (SFD) visually represent the components and their interactions in systems.
- **Purpose in Modeling:** SFD facilitates understanding of dynamic systems by illustrating quantitative changes in stocks over time.
- **SFD vs. CLD:** Unlike Casual Loop Diagrams, SFD specifically delineates stocks, flows, and their interdependencies graphically.

# Key Components of SFD

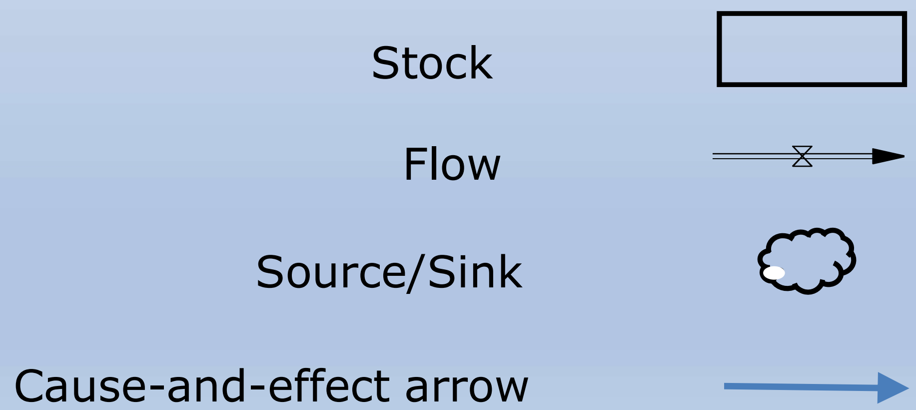
- **Stocks Overview:** Stocks represent accumulations in a system, quantified as the integral of inflows minus outflows.
- **Flow Dynamics:** Flows signify the rates of change in stocks, calculated using inflow and outflow intervals in models.
- **Auxiliary Variables:** Auxiliary variables enhance model complexity by linking stocks and flows through mathematical relationships and functions.

$$x_t = \int_{t_0}^{t_1} (u_t - y_t)\mathrm{d}t + c$$

Where: $u_t$ – input flow at time t, $t \in (t_0\ t_1)$, $y_t$ – output flow at time t, c - initial resource level at time $t_0$.

# Symbolic Representation

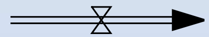
- **Stock Symbol:** Rectangles depict stocks in SFD, representing accumulations of resources that change over time.
- **Flow Symbol:** Arrows indicate flows, showing rates affecting stock levels; they are crucial for dynamic relationships.
- **Auxiliary Variable Symbol:** Dotted lines represent auxiliary variables, which influence flows and stocks through calculated relationships.



# CFD building blocks explained

LEVEL:

- AKA stock, accumulation, or state variable
- A quantity that accumulates over time
- Change its value by accumulating or integrating rates
- Change continuously over time



RATE:

- AKA flow, activity, movement
- Change the values of levels

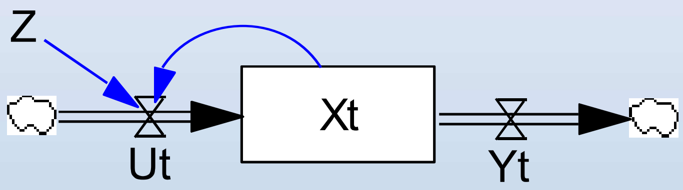
SOURCE/SINK:

- Source represents environment outside of the model
- Sink is where flows terminate outside the system

CAUSE-AND-EFFECT ARROW

# Equations in System Dynamics

- Levels are described with integral equations
- Rates are given with algebraic equations




For example:

$$U_t = Z - X_t$$

$$x_t = \int (u_t - y_t) \cdot dt + C$$

# Feedback Loop Interactions

- **Positive Feedback Loops:** Positive feedback loops amplify changes, possibly leading to exponential growth or system instability in SFD models.
- **Negative Feedback Loops:** Negative feedback loops counteract deviations, promoting stability by regulating stock levels towards desired targets.
- **System Stability Implications:** The interplay of feedback loops determines overall system stability, affecting dynamic behavior during various perturbations.

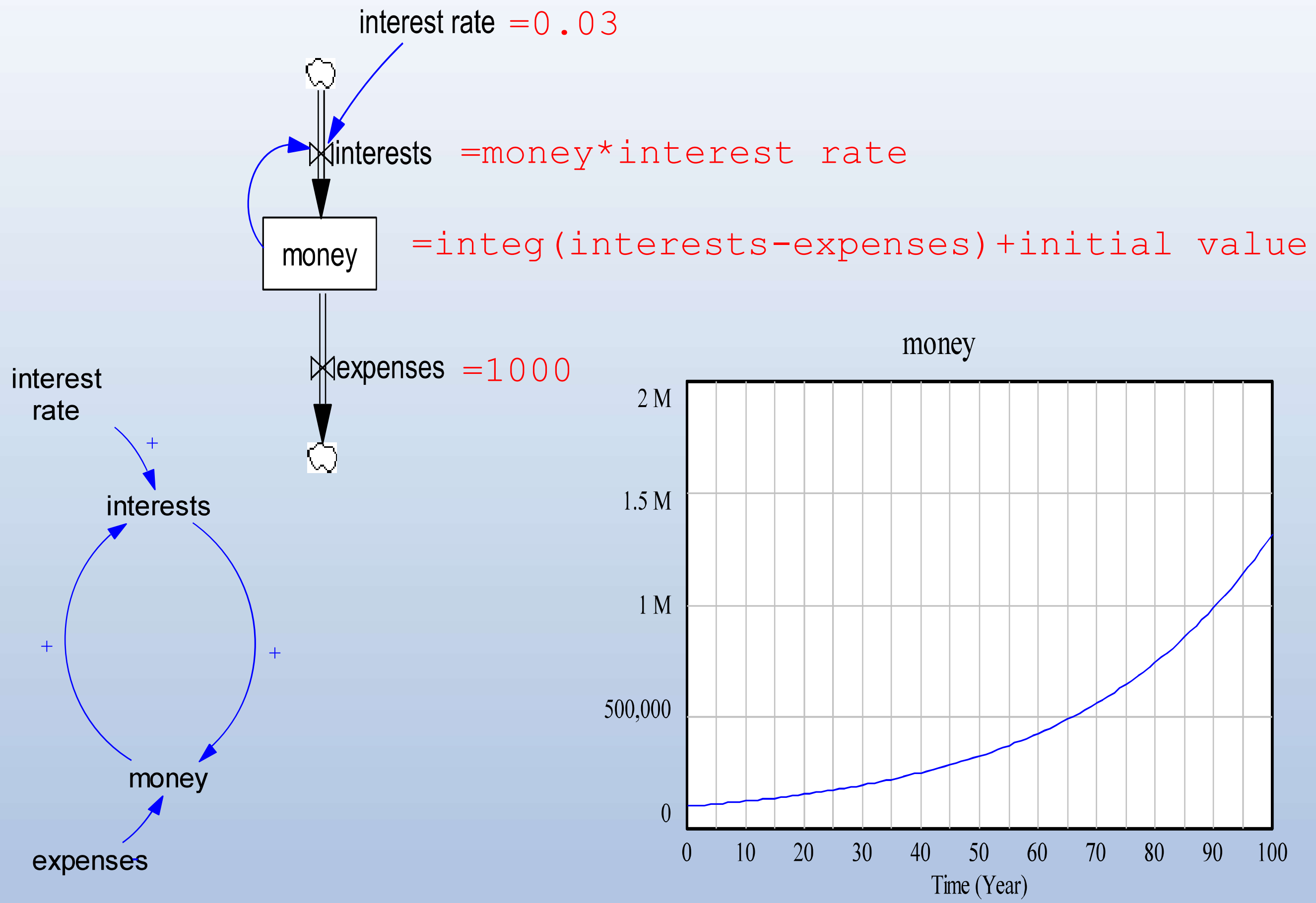
# System Archetypes

- **Growth Archetype:** Characterized by exponential increase; examples include population growth and wealth accumulation systems.
- **Goal-Seeking Archetype:** Stabilizes around a target; for instance, maintaining water levels in a reservoir is illustrative.
- **Oscillation Archetype:** Features periodic fluctuations; an example can be the seasonal fluctuation of inventory levels.

# Exponential Growth/Decay Models

- **Exponential Growth Example:** The bank account model shows capital growing exponentially via reinvested interest, illustrated mathematically.
- **Decay Model Description:** Conversely, decay can be modeled via withdrawals exceeding deposits, formulated by related differential equations.
- **Supporting Equations:** Utilizing INTEG function, growth and decay rates are expressed as Stock = INTEG(inflow - outflow).

# Growth archetype: Model of a bank account



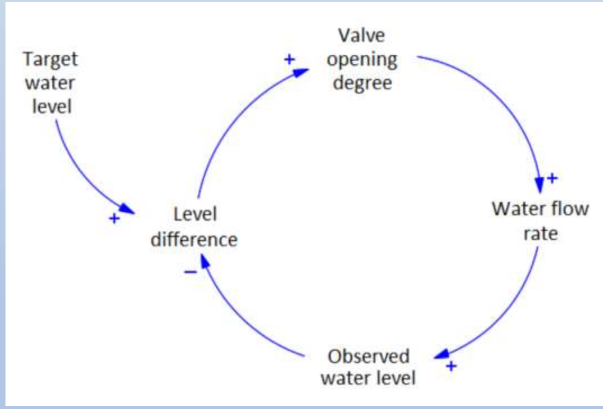

A change in the state of the system (capital) with a positive feedback loop can take the form of exponential growth or exponential decline

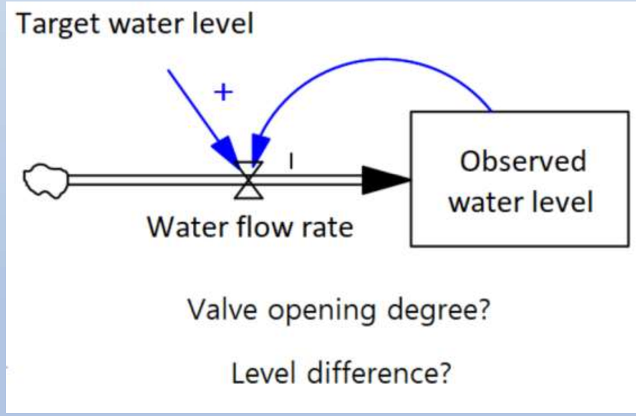
# Goal-Seeking Models

- **Feedback Mechanisms:** Feedback loops regulate water inflow and outflow, balancing the tank's water level against desired parameters.
- **Desired Level Achievement:** Maintaining target water levels involves adapting inflow rates to counterbalance variable demands and losses.
- **Dynamic Behavior Insights:** SFD models reveal interactions between tanks' inflows and outflows, demonstrating stability through feedback processes.

# Goal-seeking archetype: Model of a tank filling

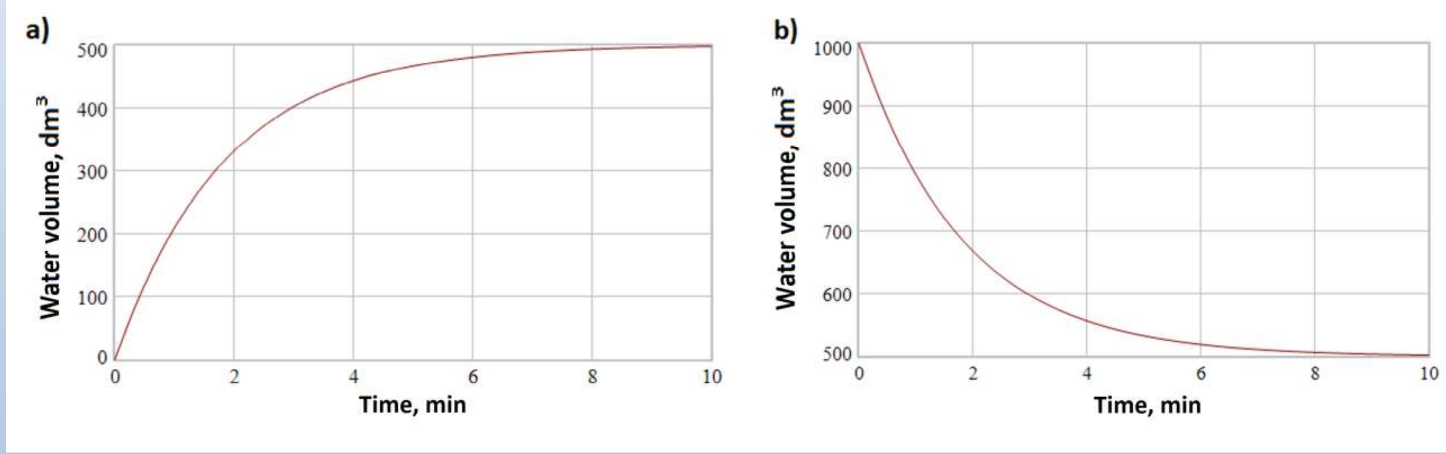
Goal-seeking behavior is characterized by systems with a dominant negative feedback loop in their structure. The operation of this loop causes the system (the values of the state variables) to stabilize at a preset level. An example is the behavior of a tank filling system. The degree of opening of the valve is proportional to the gap between the preset and actual volume of water.



# Goal-seeking archetype: Model of a tank filling

Inflow_water takes on positive values (water flows into the tank) and negative values (water flows out of the tank). The change in the volume of water in the tank was determined for two cases: a) the initial volume of water is lower than the setpoint, b) the initial volume of water is higher than the setpoint.




The water tank equation takes the form:

Volume_of_water = INTEG(Water flow)+Initial_volume)

Water flow = ( Target volume – Observed volume)/2

# Oscillatory Systems

- **Delayed Feedback Impact:** Systems with delayed feedback produce oscillations, showcasing instability and regular fluctuations over time.
- **Example of Oscillatory Behavior:** Inventory management demonstrates delays causing stock discrepancies, resulting in periodic overstocking and shortages.
- **Control Loop Analysis:** Control loops exhibiting delays can lead to oscillatory outcomes; stabilizing these requires precise timing adjustments.

# Role of Delays

- **Fixed Delay Dynamics:** Fixed delays introduce consistent time lag in responses, calculated as DELAY FIXED(input, delay_time, initial_value).
- **Distributed Delay Effects:** Distributed delays gradually modify system behavior, modeled with DELAY1 and DELAY3 functions affecting output signals.
- **Mathematical Impact:** Both delay types significantly influence stability, oscillations arise when feedback loops experience improper timing adjustments.

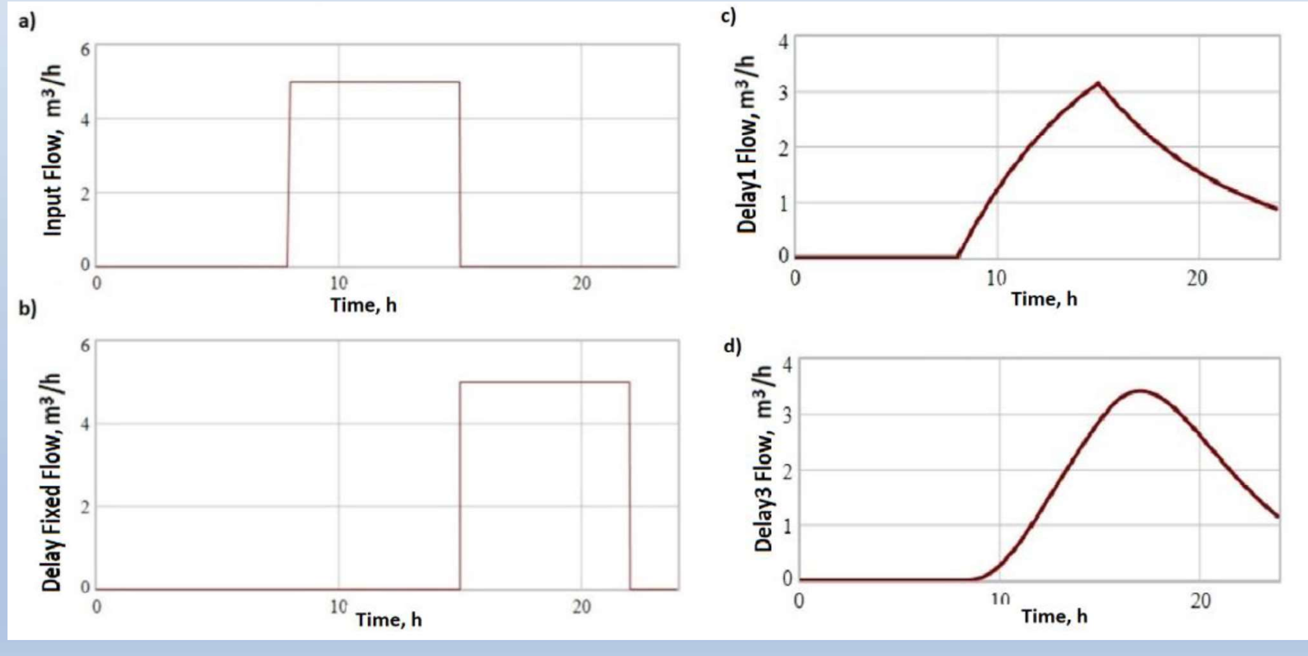
# Oscillatory Systems

Delays shift over time the change in the characteristics of the elements of the system that are in a cause-effect relationship (the effect occurs after a certain period of time after the occurrence of the cause). For time-focused delay, the DELAY FIXED function is used in the form:

***DELAY FIXED({input}, {delay time} , {initial value})***

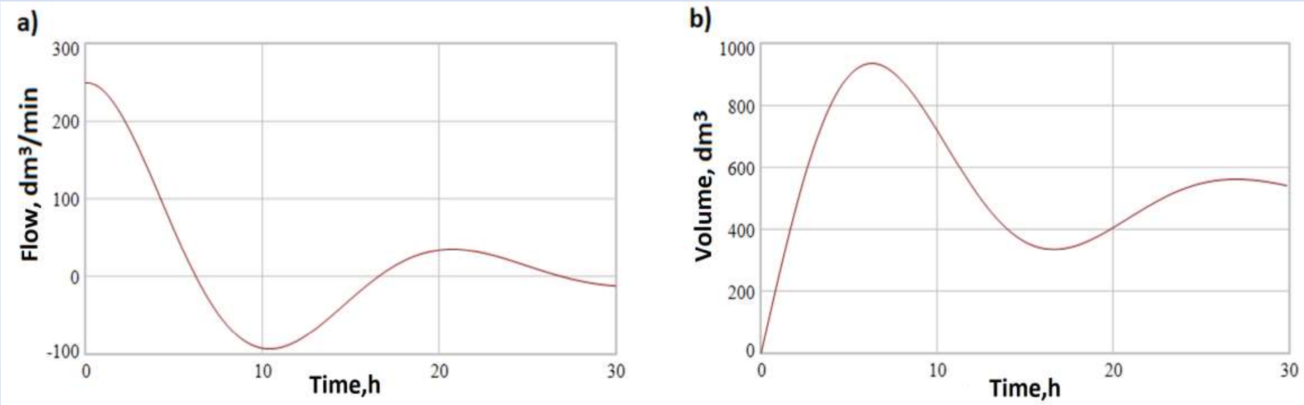
DELAY1 or DELAY3 functions are used to model the distributed delay:

***DELAYx ( {input} , {dtime} )***



# Oscillatory Systems

Delays spread over time can lead to instabilities, oscillations or overdrive. For example, the simulation results obtained for the tank filling model  show a delay in the change of the flow rate of the water supply:



# Mixed Feedback Systems

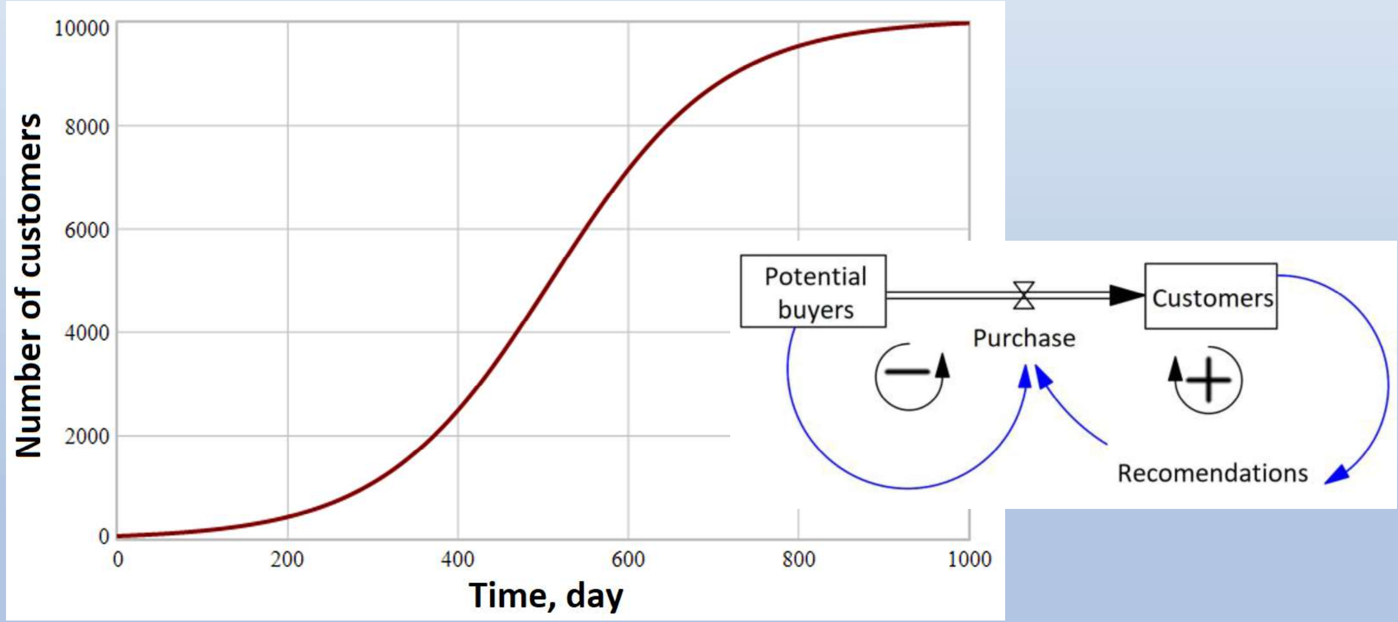
- **Complex System Behavior:** Mixed feedback loops create intricate behaviors, unpredictable dynamics often resulting from their interconnections.
- **Interdependence of Loops:** The interaction between positive and negative loops complicates modeling efforts, necessitating detailed analysis.
- **Modeling Implications:** Understanding these behaviors aids in predicting outcomes and optimizing system responses during simulations.

# S-Shaped Growth Models

- **S-shaped Growth Dynamics:** S-shaped growth patterns represent gradual increase, stabilizing as resources reach carrying capacity in systems.
- **Real-world Applications:** Examples include market penetration and product adoption curves, illustrating S-shaped trends in business dynamics.
- **Illustrative Diagrams:** Diagrams depict gradual growth transitions in ecosystems or product lifecycles, emphasizing stages of development.
- S-type growth is caused by a nonlinear interaction between positive and negative feedback loops and occurs when conditions are met:
  - the growth limit to which the system state is moving is specified,
  - the initial state of the system is far from the limit of its growth,
  - the stabilizing feedback loop does not contain delays.

# S-Shaped Growth: Acquiring new clients

Positive recommendations lead to new customers. More customers lead to more recommendations and again result in more customers. This growth is initially exponential in nature. However, there is a limit to the growth of the number of customers which can be expressed by the principle: “The more customers a company has, the fewer potential buyers are left to acquire.” Following this limitation, after the initial exponential increase, the customer population grows more and more slowly and stabilizes at a fixed level.



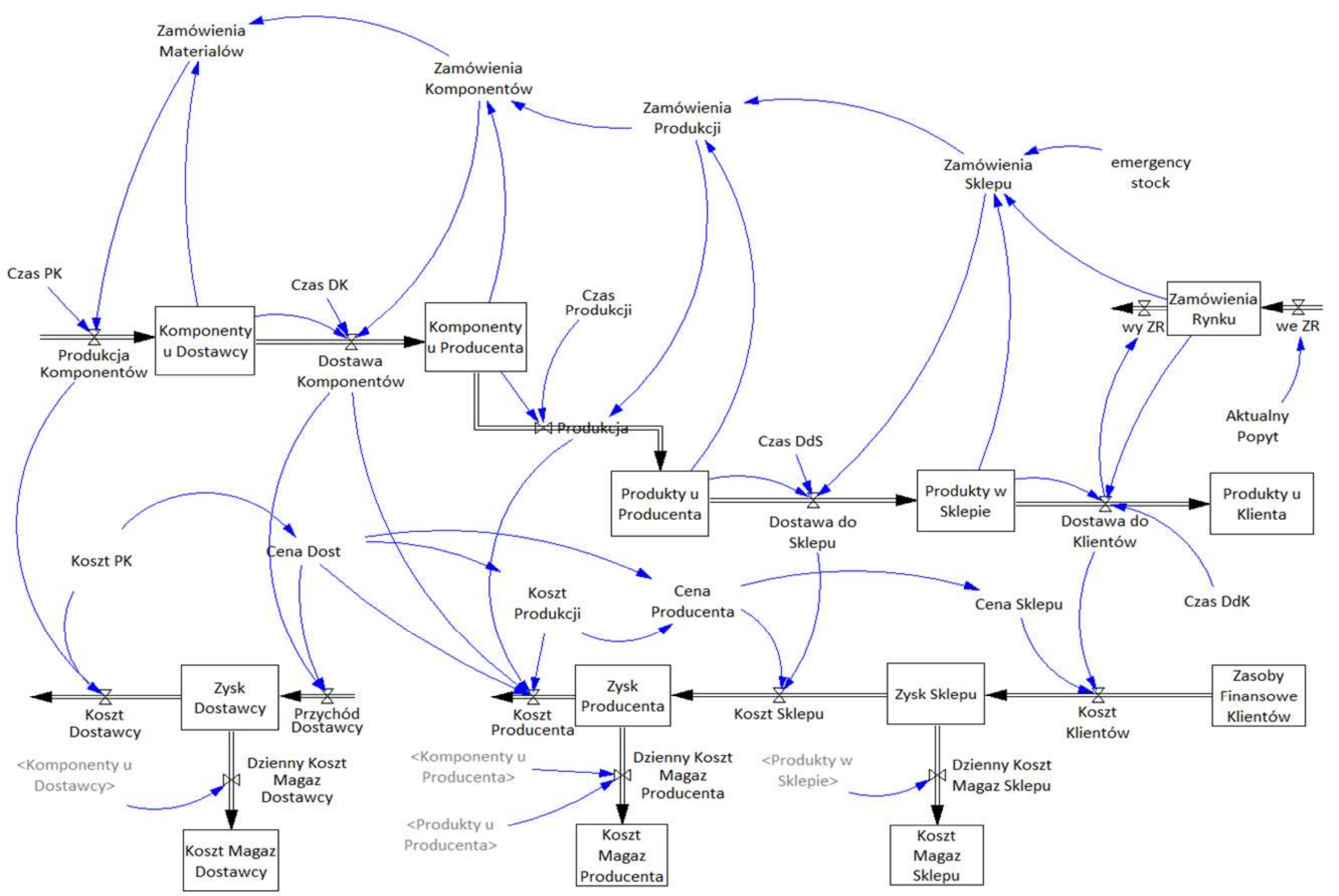
# Practical Implementation of SFD

- **Business Applications:** SFD analyzes inventory systems, optimizing stock levels and mitigating shortages through flow management strategies.
- **Environmental Modeling:** In environmental science, SFD models natural resource management, particularly in tracking water or energy flows.
- **Public Health Scenarios:** SFD aids in understanding disease spread dynamics, enabling effective resource allocation based on population needs.

# Case Studies

- **Manufacturing:** SFD utilized in manufacturing to streamline production processes, improving efficiency and reducing lead times significantly.
- **Healthcare Applications:** Modeling patient flow in hospitals using SFD resulted in optimized resource allocation, enhancing service delivery effectively.
- **Sustainable Resource Management:** In environmental contexts, SFD helped manage water resources, balancing consumption with conservation efforts for sustainability.

# Supply chain model



# Industry Applications

- **Production Efficiency:** SFD streamlines production processes, optimizing equipment usage and minimizing idle times in manufacturing systems.
- **Financial Forecasting:** In finance, SFD models cash flow dynamics, enabling projections for investment returns and risk management.
- **Logistics Management:** SFD aids logistical operations by tracking inventory flows, ensuring timely deliveries and reducing stock shortages.

# Software Tools Overview

- **Vensim Overview:** Vensim is a powerful software tool designed for modeling system dynamics using Stocks and Flows Diagrams.
- **Key Features:** It offers functionalities like dynamic simulations, sensitivity analysis, and user-friendly graphical interfaces for SFD creation.
- **Modeling Capabilities:** Users can construct complex models with intuitive drag-and-drop elements, facilitating better understanding of interactions.

# Mathematical Models

- **Integral Formulations in SFD:** Integral equations define stock levels, calculated through the difference of inflows and outflows over time.
- **Dynamic Simulation Equations:** Simulation relies on differential equations, allowing for modeling changing systems across various scenarios effectively.
- **Parameter Sensitivity Analysis:** Sensitivity analyses evaluate how changes in parameters influence outcomes within SFD models, aiding decision-making.

# Advanced SFD Features: Mathematical functions

- **Logical Functions in SFD:** Utilizing logical functions enhances decision-making protocols, incorporating conditional relationships in system dynamics models.
- **Mathematical Functions Integration:** Mathematical functions like IF THEN ELSE provide essential control over variable interactions within dynamic systems.
  - *IF THEN ELSE( {cond} , {ontrue} , {onfalse} )*
- **Conditional expressions** allow you to enter criteria connected by logical operators, such as AND, OR. The standard variable "Time" is used to store the simulation time.
  - *Processing = IF THEN ELSE( Time ≥ 7.5 :AND: Time ≤ 14.5 , $Processing_{max}$ 1, 0 )*

# Advanced SFD Features: Mathematical functions

- The MODULO function determines the remainder of the quotient of two integers. It is used, for example, to define the execution time of periodically repeating operations and takes the form:
  - ***MODULO( {x} , {base} ), Where: x - divisor, base - divider.***
- The Delivery flow using the MODULO function takes the form:
  - ***Delivery = IF THEN ELSE(MODULO(Time, 24) = 12, 1 , 0 )***
- Functions generating pseudo-random values are introduced to determine the model's response to disturbances
  - ***RANDOM UNIFORM({min}, {max}),***
- The RANDOM UNIFORM function used to generate random demand values in the range of values <500, 700> takes the form:
  - ***Demand = RANDOM UNIFORM(500, 700)***

# Visualization Tips

- **Simplify Diagrams:** Limit complexity by using clear symbols, straightforward connections, and minimal text for easy comprehension.
- **Consistent Labeling:** Ensure all components are consistently labeled for clarity, enhancing stakeholder understanding of system dynamics.
- **Use Color Coding:** Employ color coding to differentiate stocks, flows, and auxiliary variables, improving visual representation and impact.

# Challenges in SFD Modeling

- **Key Challenges in SFD Modeling:** Modeling complexity can lead to misinterpretations; clarity is essential for effective communication and analysis.
- **Proposed Simplification Strategies:** Employ user-friendly software tools and standardized symbols to enhance model usability and stakeholder engagement.
- **Training and Knowledge Sharing:** Implement training programs for stakeholders, improving understanding of SFD structures and fostering collaboration.

# Summary

- **Importance of SFD Modeling:** SFD modeling enables comprehensive understanding of complex systems, facilitating effective decision-making and problem-solving.
- **Future Research Areas:** Emerging trends include integrating AI for predictive analytics, enhancing model accuracy and operational efficiencies.
- **Adapting to Challenges:** Ongoing challenges necessitate advances in user training programs to promote stakeholder engagement in SFD applications.

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