Logic systems

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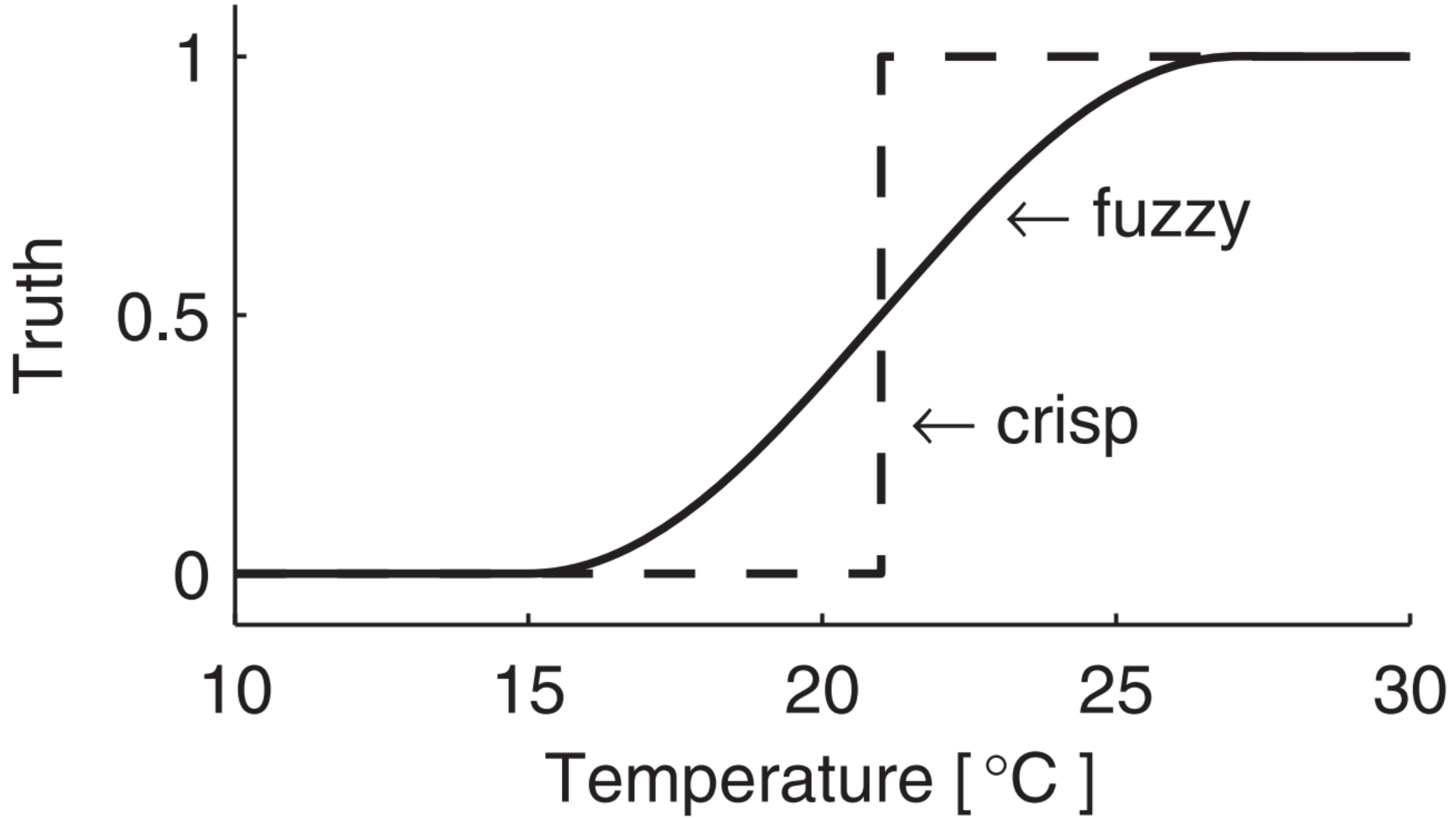
Lecture 02

Comparison between crisp and fuzzy set



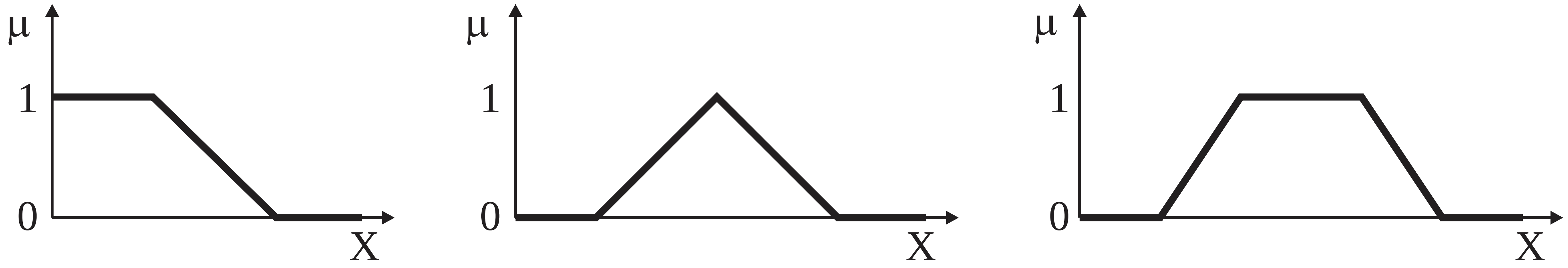
Characteristic function of a fuzzy set

- $\mu_A : X \rightarrow [0, 1]$

Some researchers

- Lotfi Aliasker Zadeh
- Ebrahim (Abe) H. Mamdani
- Michio Sugeno

Typical examples of fuzzy sets



Union of two fuzzy sets

- C = A ∪ B
- $\mu_C(x) = \max[\mu_A(x), \mu_B(x)]$
- T-norms

algebrajska vsota (angl. algebraic sum)	$\mu_A(x) + \mu_B(x) - \mu_A(x) \cdot \mu_B(x)$
omejena vsota (angl. bounded sum)	$\min(\mu_A(x) + \mu_B(x), 1)$
izrazita vsota (angl. drastic sum)	$\begin{cases} \mu_B(x), & \mu_A(x) = 0 \\ \mu_A(x), & \mu_B(x) = 0 \\ 0, & \text{v vseh drugih primerih} \end{cases}$

Intersection of two fuzzy sets

- C = A ∩ B
- $\mu_C(x) = \min [\mu_A(x), \mu_B(x)]$
- S-norms (T-co-norms)

algebrajski produkt (angl. algebraic product)	$\mu_A(x) \cdot \mu_B(x)$
omejeni produkt (angl. bounded product	$\max(\mu_A(x) + \mu_B(x) - 1, 0)$
izraziti produkt (angl. drastic product	$\mu_B(x)$, $\mu_A(x) = 1$ $\mu_A(x)$, $\mu_B(x) = 1$ 0, v vseh drugih primerih

Complement

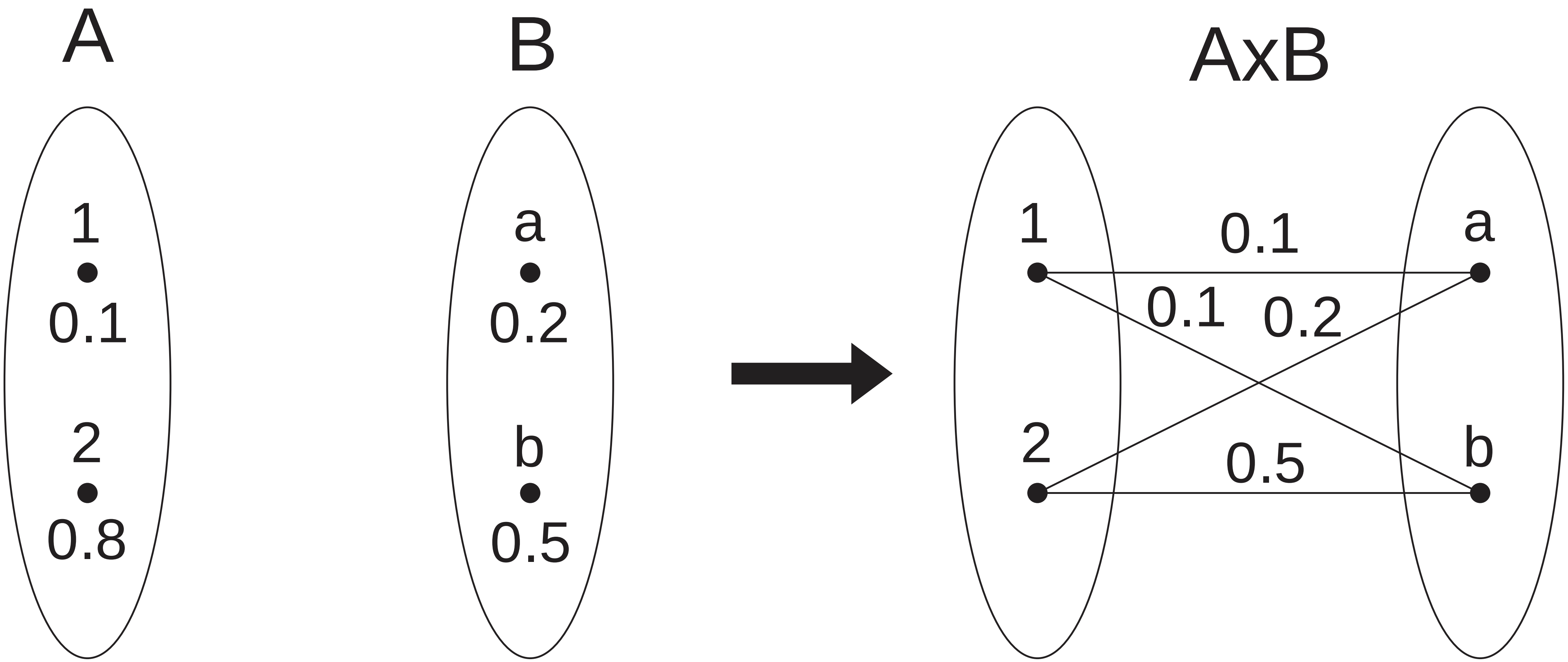
- Relative complement of the fuzzy set A in relation to the fuzzy set B
- E=B-A
- $\mu_E(x) = \max [0, \mu_B(x)-\mu_A(x)]$

- In case the set B is the universal set X, we get: $\mu_{\bar{A}}(x) = 1 - \mu_A(x)$

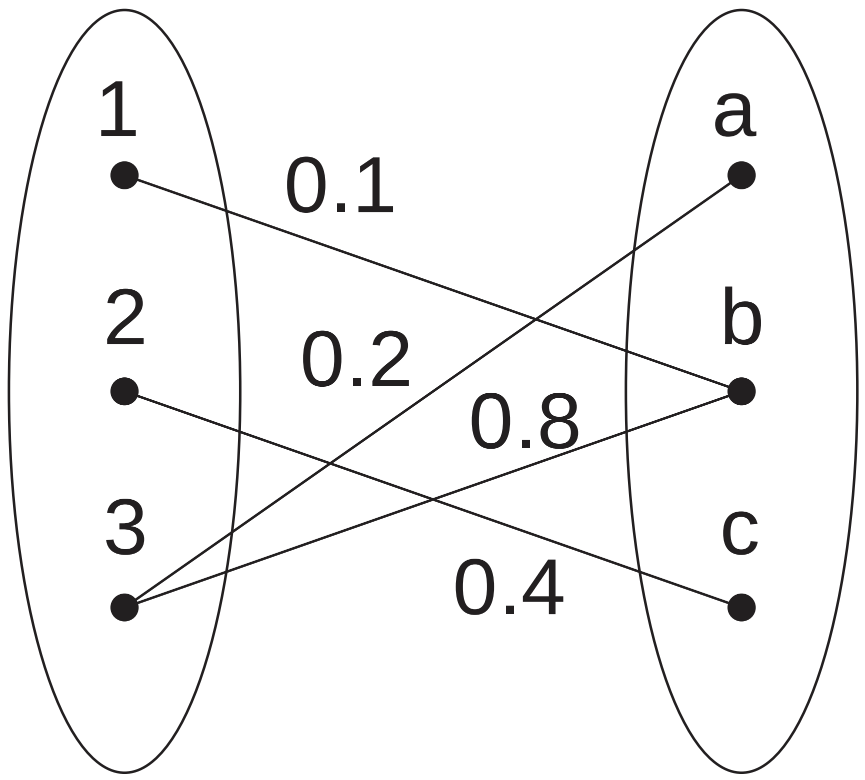
Fuzzy relations

- Crisp relations
- $R \subset X \times Y = \{(x, y) \mid x \in X, y \in Y\}$
- $\mu_R : X \times Y \rightarrow \{0, 1\}$
- Fuzzy relations:
- $\mu_R : X \times Y \rightarrow [0, 1]$
- Let $C = A \times B$ then: $\mu_C(x, y) = \min(\mu_A(x), \mu_B(y))$

Cartesian product on fuzzy sets



Fuzzy relations as subsets (graphically)

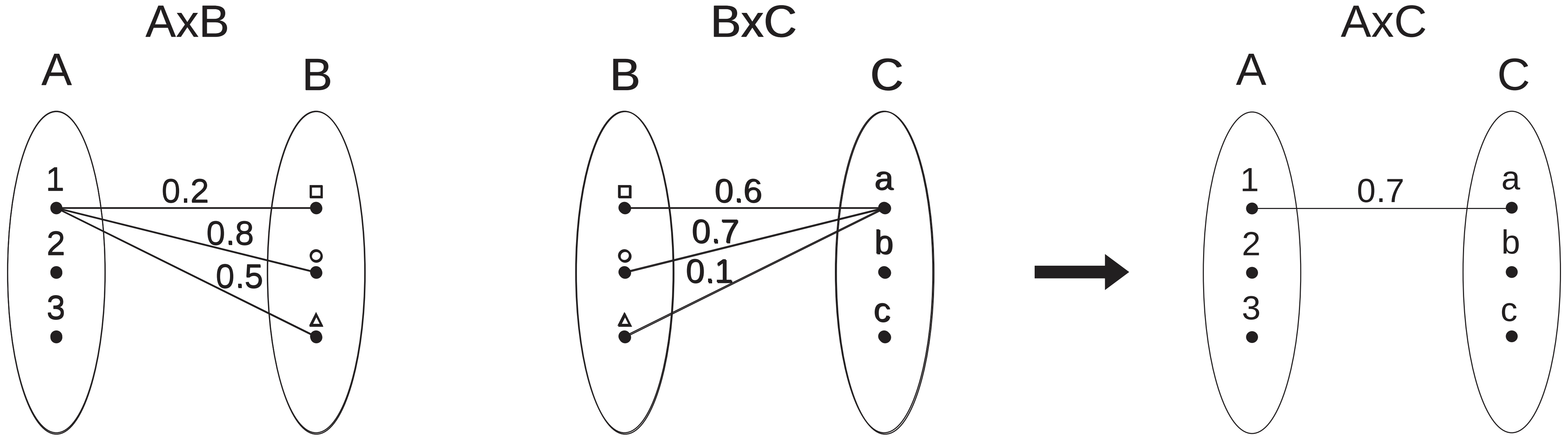


Composition of relations

- Let C and D be two fuzzy relations, which are subsets of X × Y and Y × Z, respectively. If E is composed of these two relations, that is E=C ∘ D, then:

$$\mu_E(x, z) = \max_{y=Y} (\min(\mu_C(x, y), \mu_D(y, z))$$

Composition of relations (graphically)

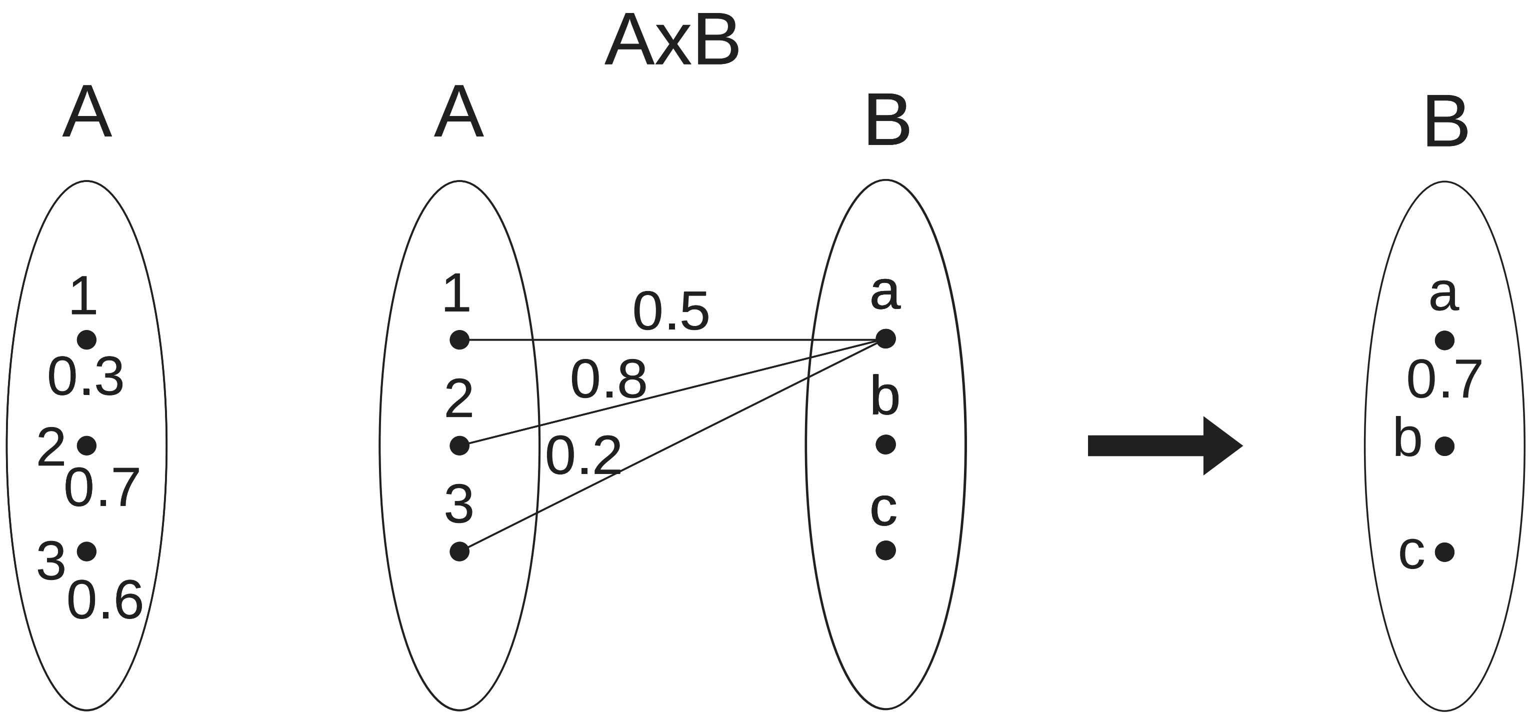


Special case

- Special case is a case when one of the relations is substituted with a fuzzy set. Let's say that we have a fuzzy set $A \subset X$ and fuzzy relation $C \subset X \times Y$. Their composition is then

$$\mu_B(y) = \max_{x=X} \left(\min(\mu_A(x), \mu_C(x,y))\right)$$

Special case graphically



Fuzzy propositions

- Conjunction: $p \wedge q \;\Rightarrow\; P \cap Q$
- Disjunction: $p \vee q \;\Rightarrow\; P \cup Q$
- Negation: $\neg p \;\Rightarrow\; \overline{P}$
- Implication: $p \Rightarrow q \;\Rightarrow\; \overline{P} \cup Q$

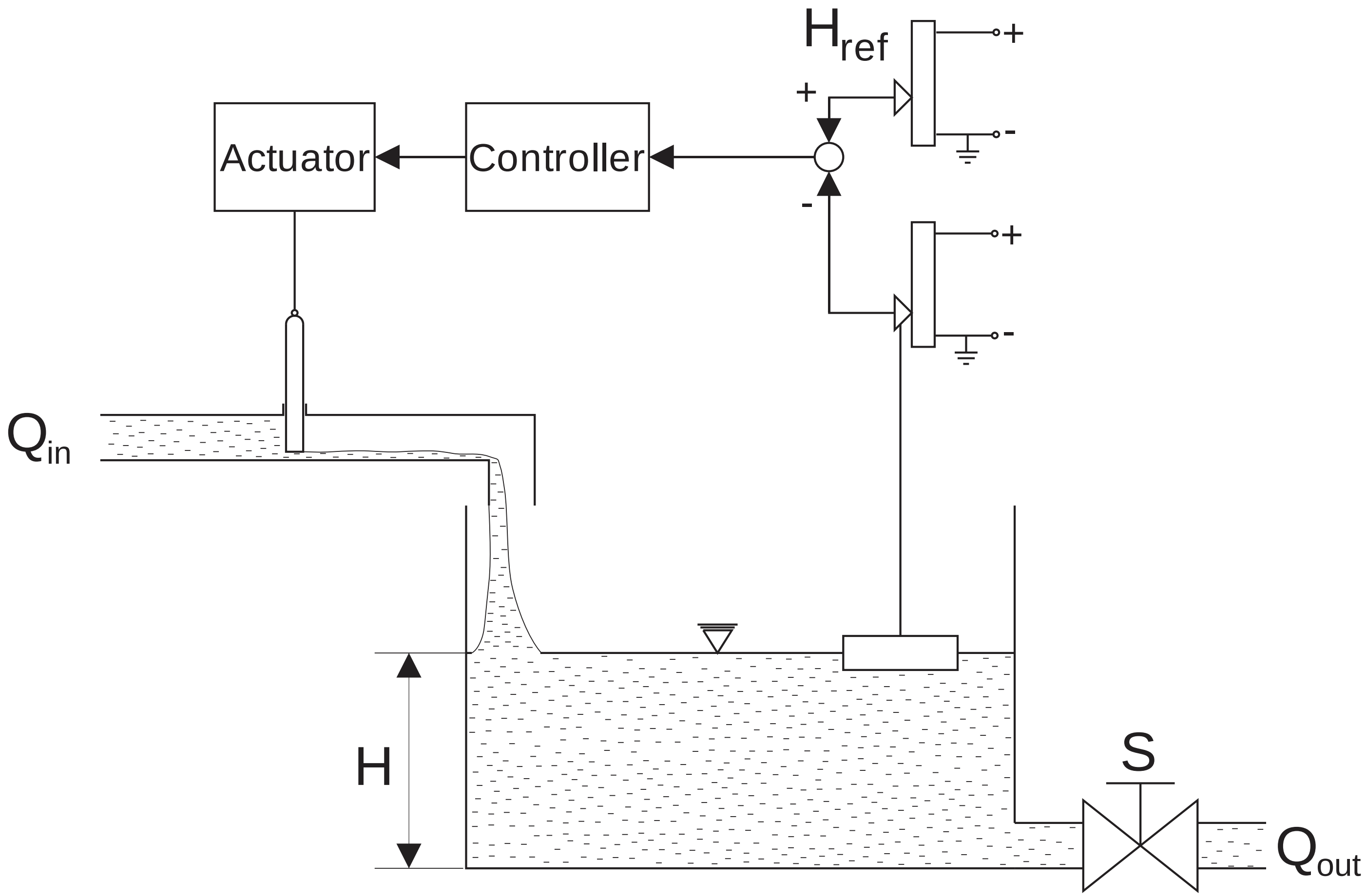
p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

p	q	$\neg p$	$\neg p \vee q$	$p \to q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Modus ponens

- $p \land (p \Rightarrow q) \Rightarrow q$
- Truth (logical) table

Fuzzy control



Fuzzy rules

Sensor	Controller	Actuator
p	$p \Rightarrow q$	q
Water level too high	If water level is too high, close the valve	Close the valve

Extension of modus ponens

- What to do if water level is high?
- Close the valve slightly
- How can we denote that in logical notation?
- $p' \wedge (p \Rightarrow q) \Rightarrow q'$
- How can we write that using sets?
- $Q' = P' \cap (\bar{P} \cup Q)$

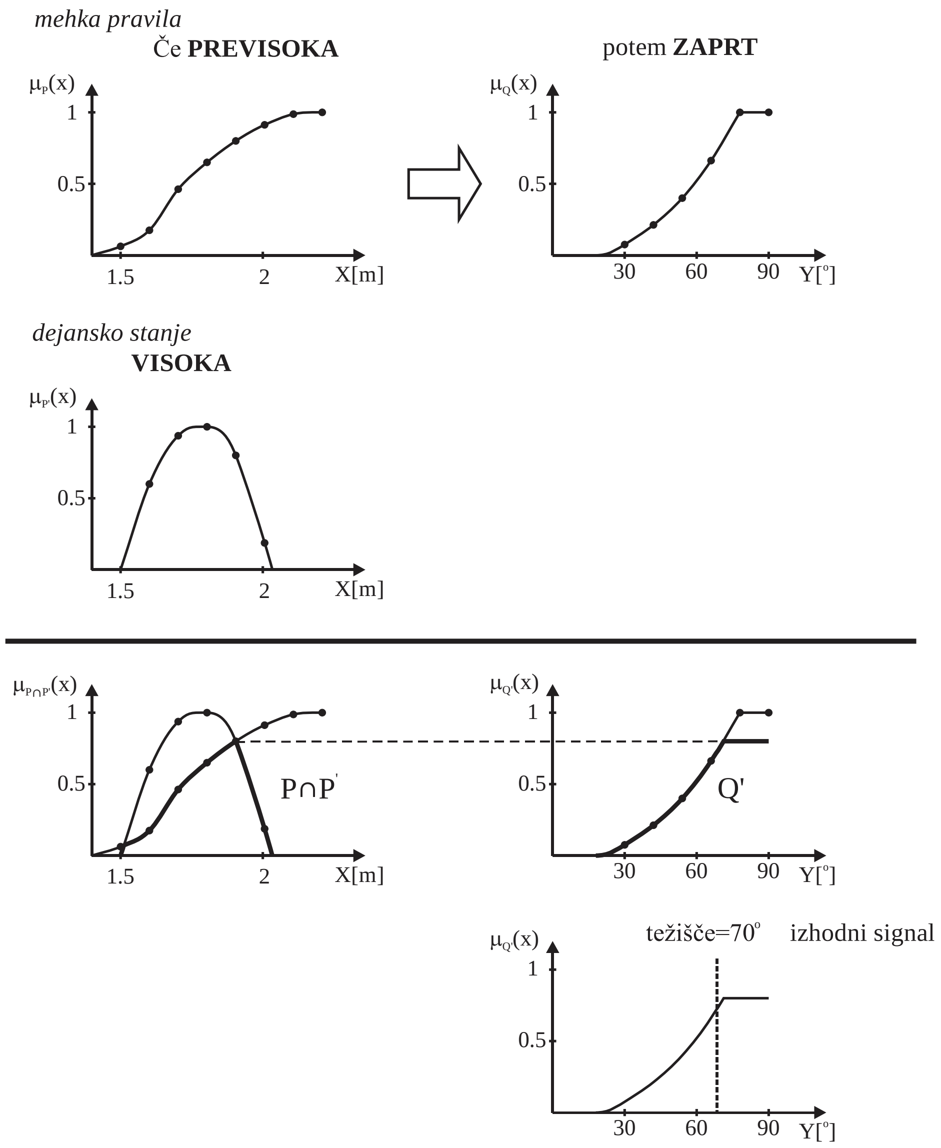
Problem and solution

- $\bar{P}$ ∪ Q doesn't make sense
- P ∩ Q should be used (Mamdani)

Final rule deduction

- Rule: $\mu_{Q'}(y) = \max_{x \in X} (\min(\mu_{P'}(x), \mu_{P \cap Q}(x, y))) = \max_{x \in X} (\min(\mu_{P'}(x), \mu_P(x), \mu_Q(y)))$
- We use equality: $\min(\mu_{P'}(x), \mu_P(x), \mu_Q(y)) = \min\left(\min_{x \in X}(\mu_{P'}(x), \mu_P(x)), \mu_Q(y)\right)$
- And finally get: $\mu_{Q'}(y) = \max_{x \in X} (\min(\mu_{P' \cap P}(x), \mu_Q(y)))$

Final rule graphically



Defuzzification

- Q' is not an appropriate output of a controller
- Center of gravity: $T = \frac{\int_Y \mu_{Q'}(y) \cdot y \, dy}{\int_Y \mu_{Q'}(y) \, dy}$

Input signal as a number

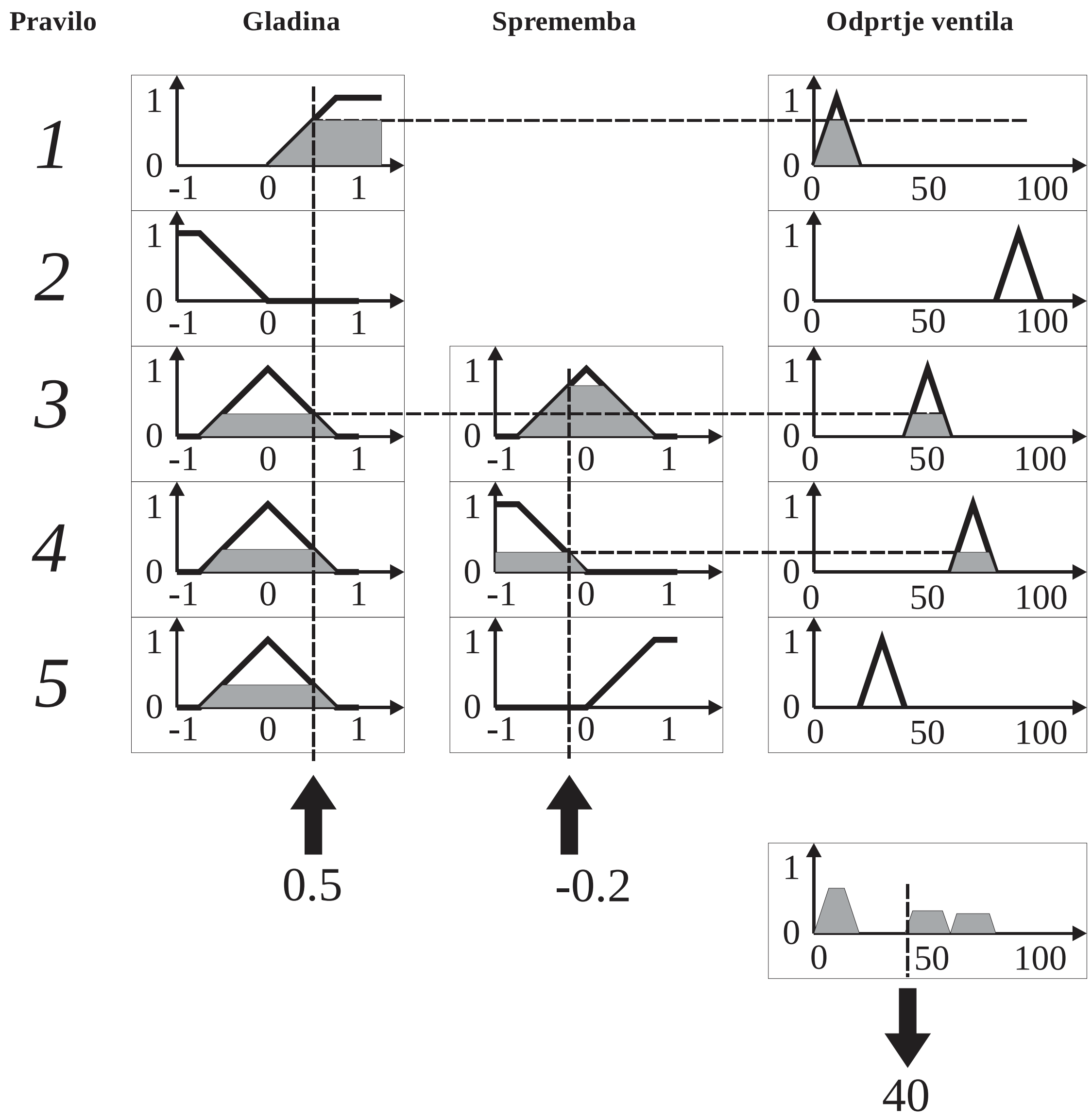
- The input signal to a controller is typically also an error signal (single value)
- The associated fuzzy set can be defined as: $\mu_{P'}(x) = \begin{cases} 1; & x = x_0 \\ 0; & x \neq x_0 \end{cases}$

A more realistic example

- A rule that makes more sense: “If the water level is high and increasing then open the valve”
- So we get a set of rules:

Št.	Gladina	Sprememba	Odprtje zasuna
1	nizka		popolnoma odprt
2	visoka		zaprt
3	normalna	nic	polovicno odprt
4	normalna	pada	75% odprt
5	normalna	visoka	25% odprt

Result (graphically)



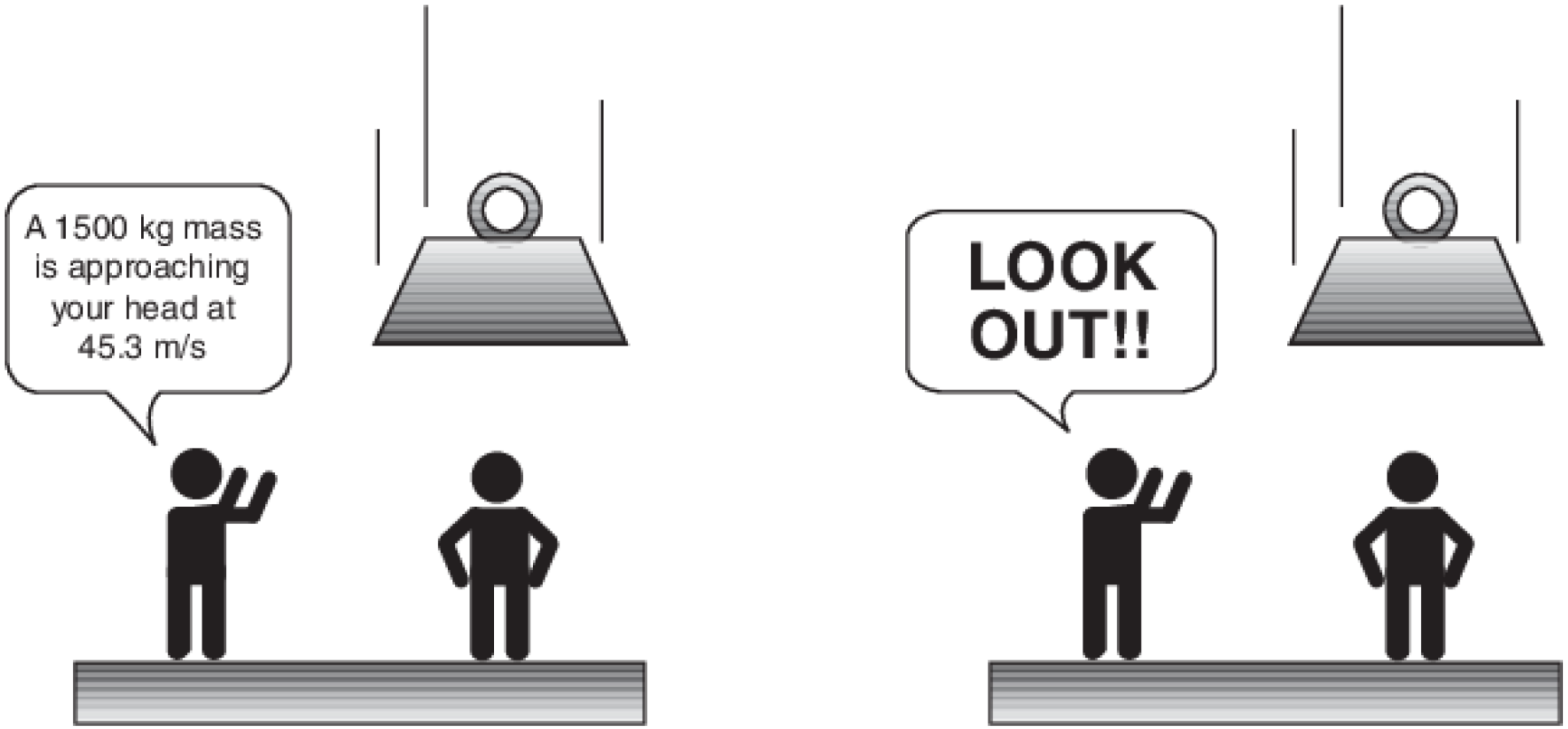
Why to use fuzzy logic

- Since the control strategy consists of if–then rules, it is easy for a process operator to read. The rules can be built from a vocabulary containing everyday words such as ‘high’, ‘low’, and ‘increasing’. Process operators can embed their experience directly.
- The fuzzy controller can accommodate many inputs and many outputs. Variables can be combined in an if–then rule with the connectives “and” and “or”. Rules are executed in parallel, implying a recommended action from each. The recommendations may be in conflict, but the controller resolves conflicts.

Why not to use fuzzy logic

- The PID controller is well understood, easy to implement – both in its digital and analogue forms – and it is widely used. By contrast, the fuzzy controller requires some knowledge of fuzzy logic. It also involves building arbitrary membership functions.
- The fuzzy controller is generally nonlinear. It does not have a simple equation like the PID, and it is more difficult to analyse mathematically; approximations are required, and it follows that stability is more difficult to guarantee.
- The fuzzy controller has more tuning parameters than the PID controller. Furthermore, it is difficult to trace the data flow during execution, which makes error correction more difficult.

Precision and significance in the real world



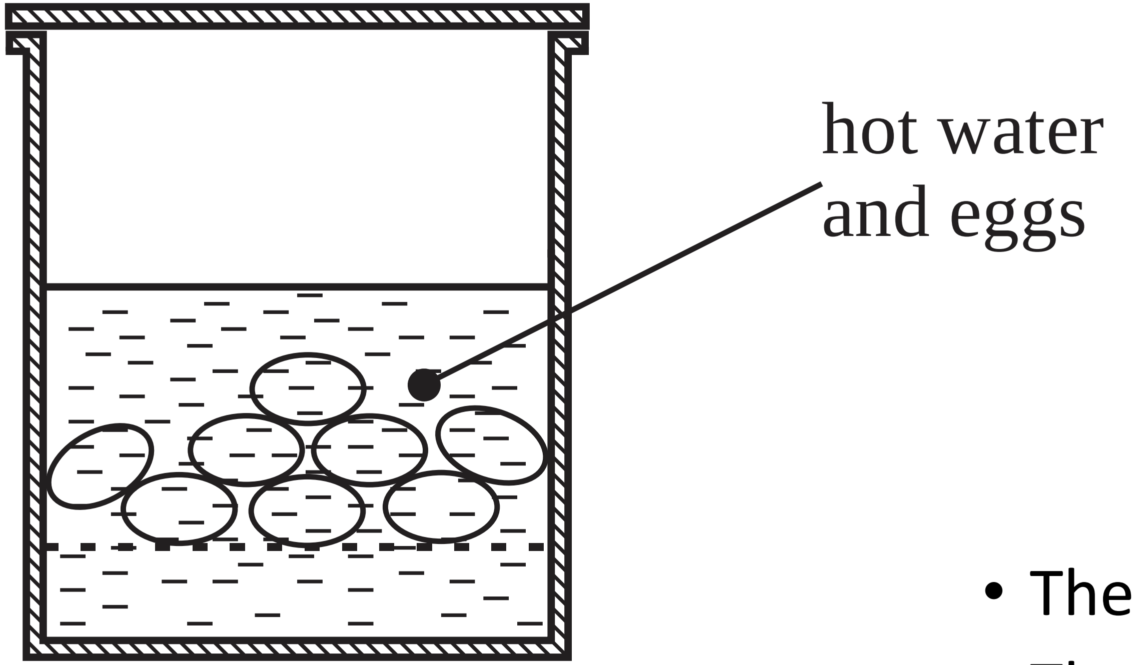
Precision

Significance

Summary

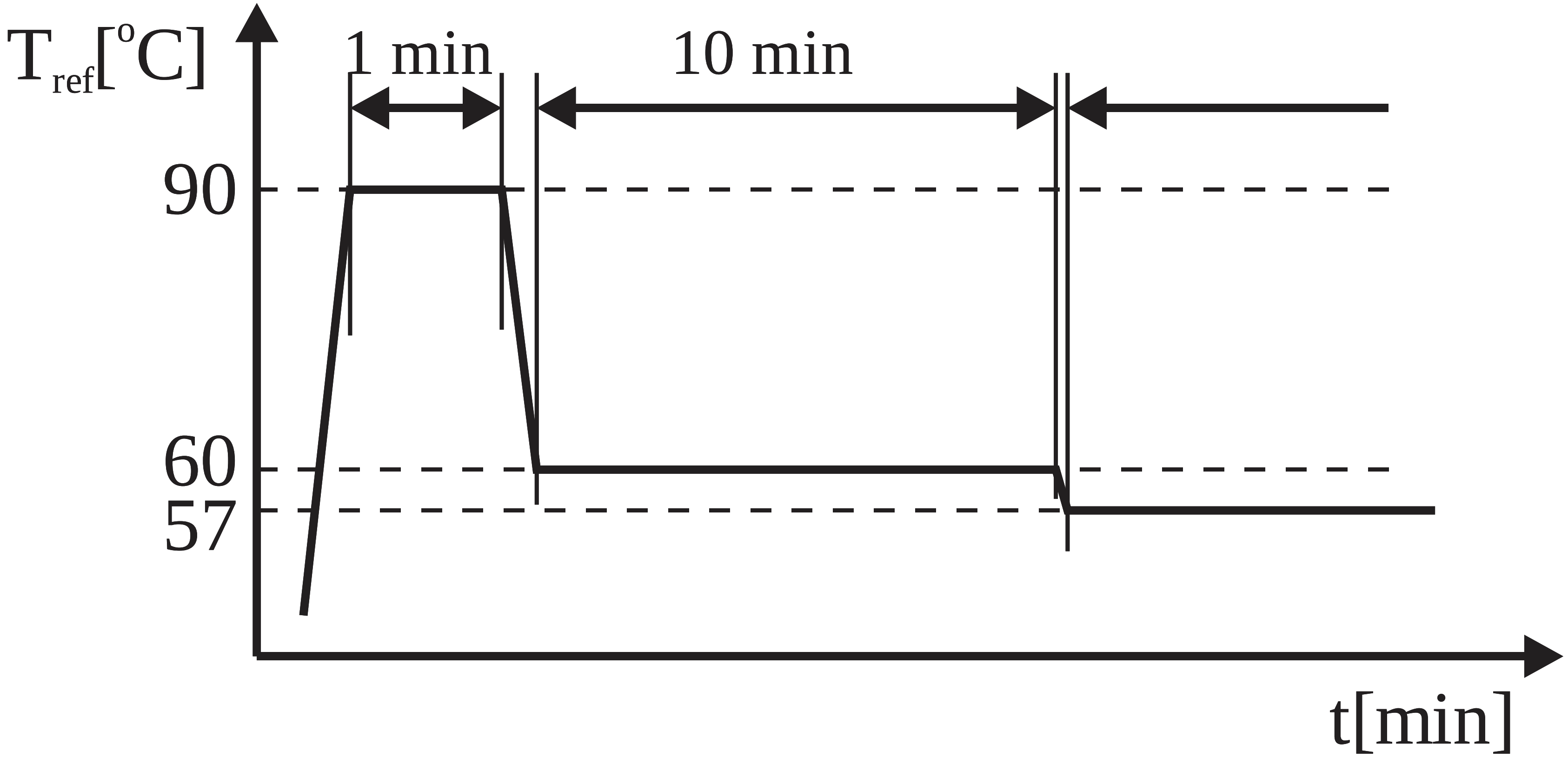
- Lofti Zadeh quote: “In almost every case you can build the same product without fuzzy logic, but fuzzy is faster and cheaper.”

Nonlinear control (fuzzy logic)

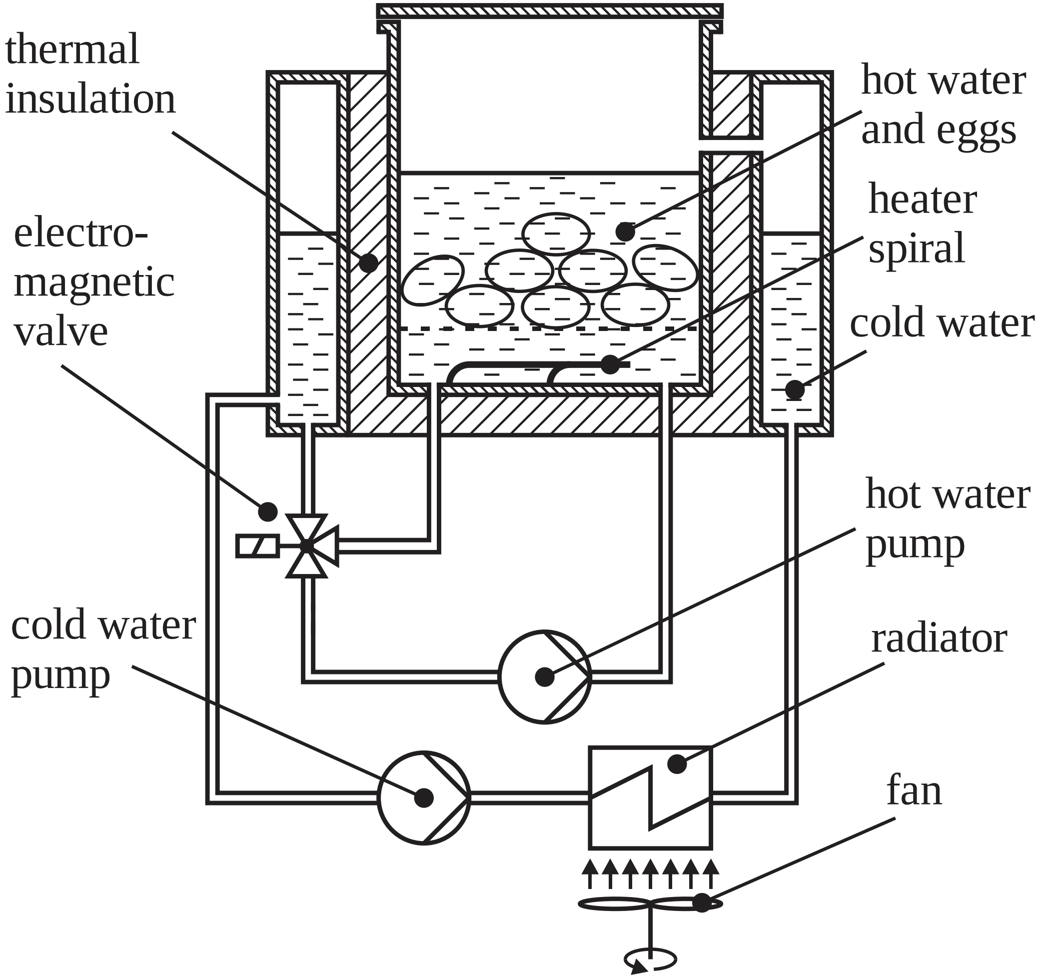


- The egg white must coagulate
- The egg yolk should not coagualte
- The eggs should be pasteurized

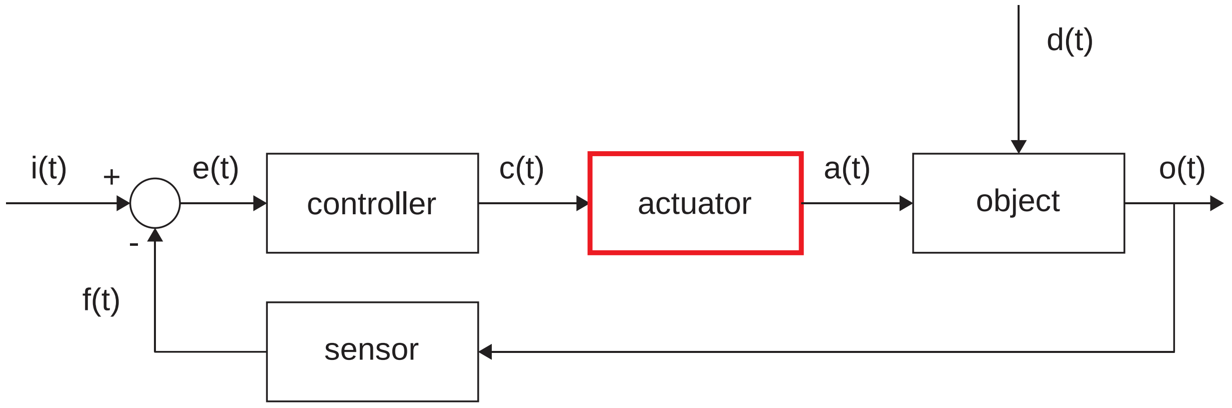
System output (water temperature) requirements

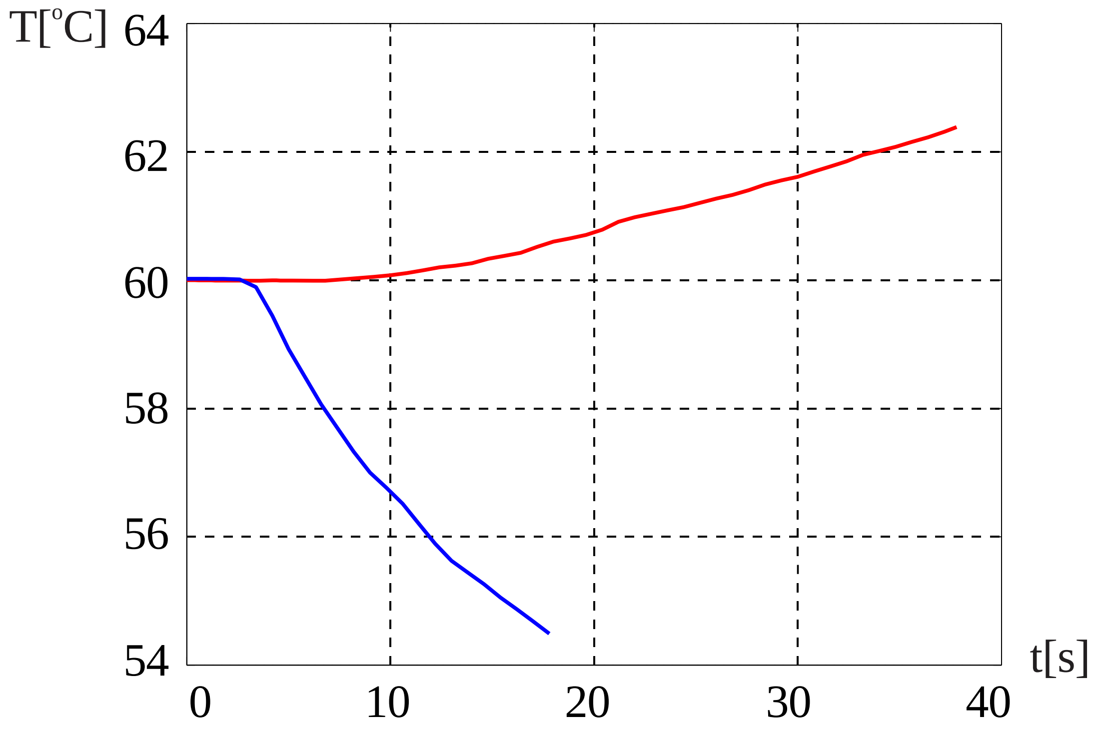


System setup



Actuator problem

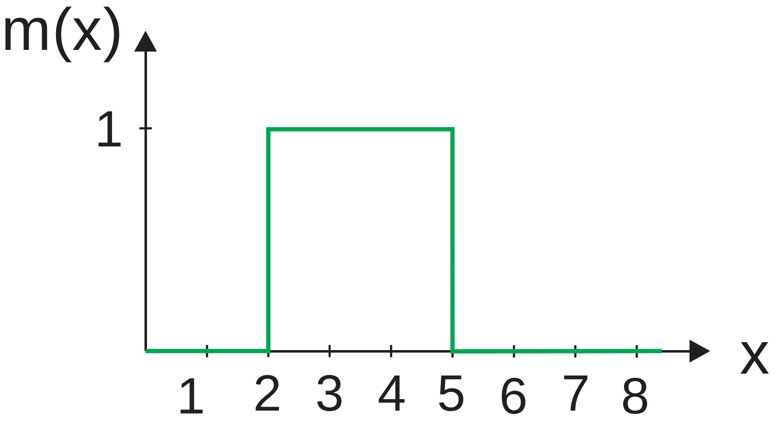




Crisp vs. Fuzzy sets

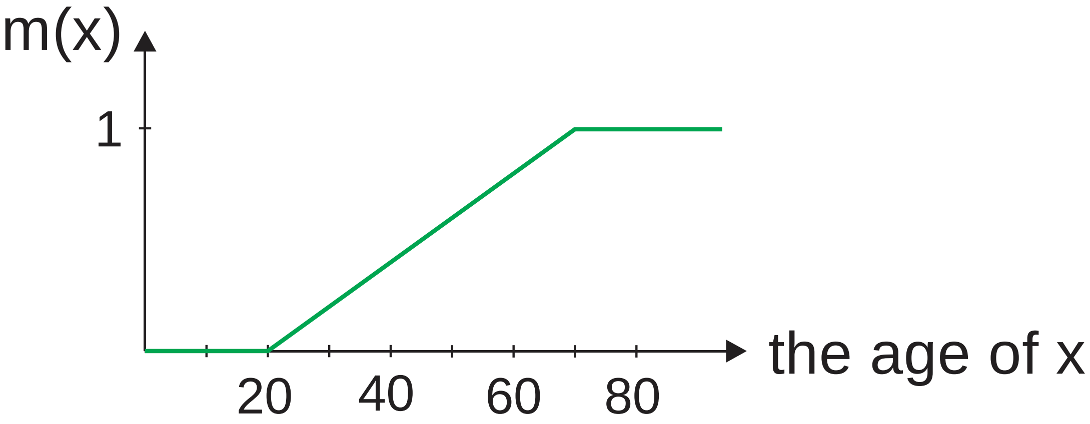
Crisp set

2<x<5

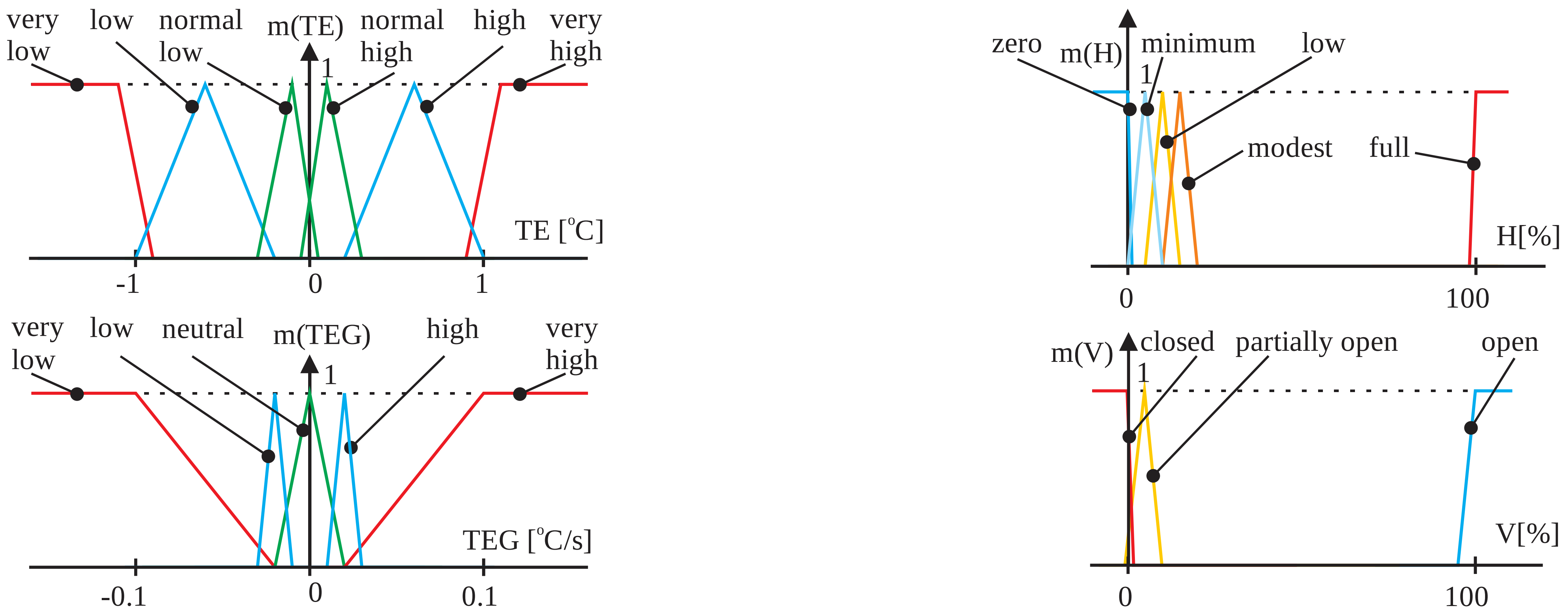


Fuzzy set

x is old



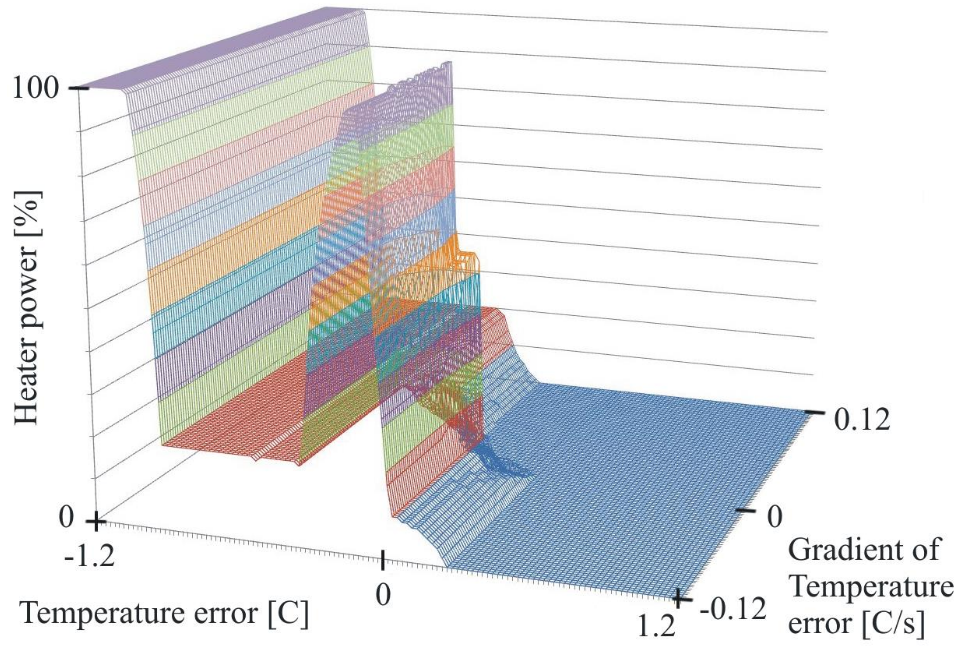
Fuzzy sets

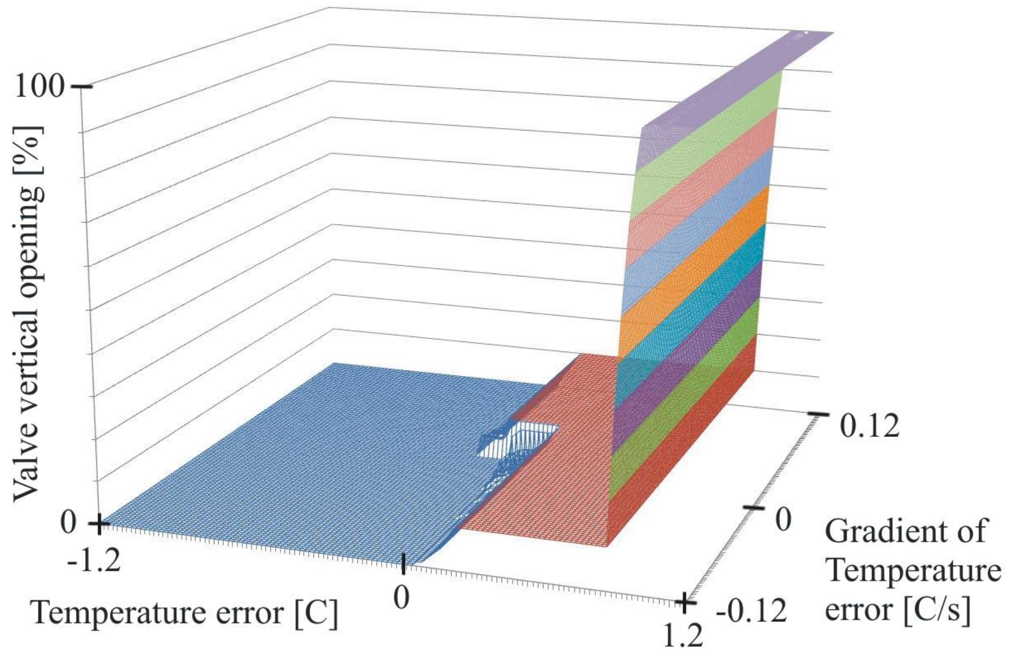

very low
low
normal low
m(TE)
normal high
high
very high
1
TE [°C]
-1
0
1
very low
low
neutral
m(TEG)
high
very high
1
TEG [°C/s]
-0.1
0
0.1
zero
m(H)
minimum
low
1
modest
full
H[%]
0
100
m(V)
closed
partially open
open
1
V[%]
0
100

Fuzzy rules

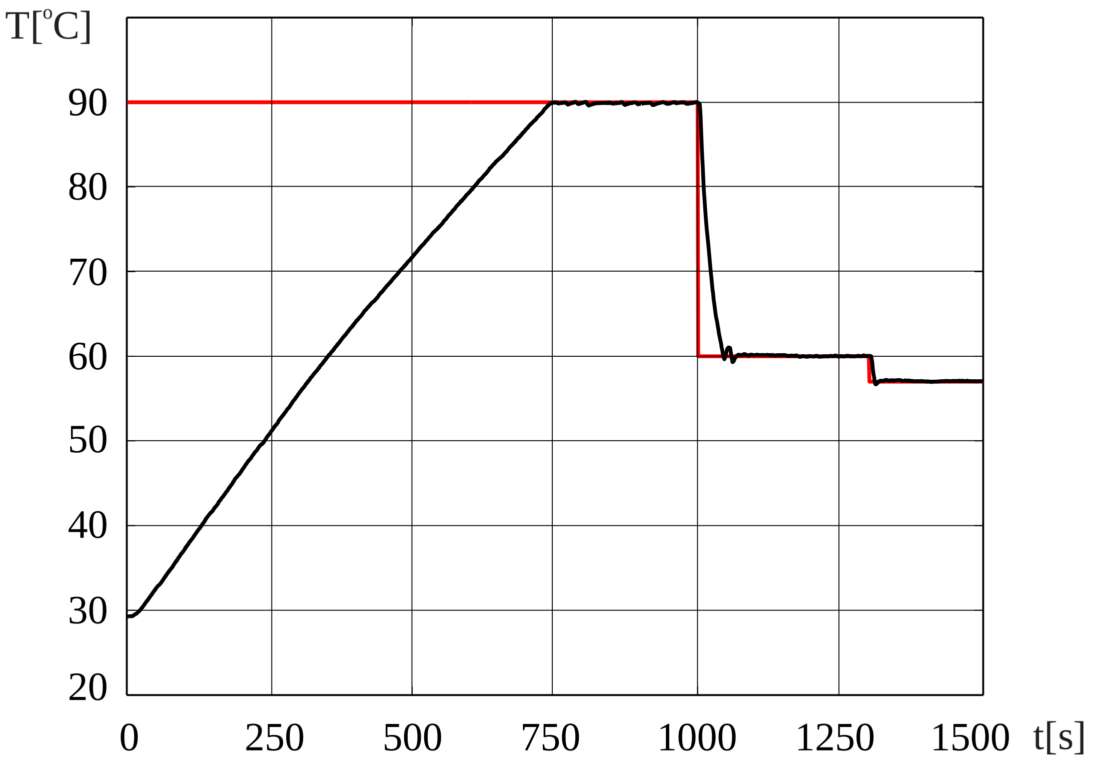
Rule number	Temperature	Temperature derivative	Heater power	Valve opening
1	very low		full	closed
2	low		modest	closed
3	normal low	very low	full	closed
4	normal low	low	modest	closed
5	normal low	neutral	low	closed
6	normal low	high	minimum	closed
7	normal low	very high	zero	closed
8	normal high	very low	low	closed
9	normal high	low	minimum	closed
10	normal high	neutral	zero	closed
11	normal high	high	zero	closed
12	normal high	very high	zero	partially open
13	high		zero	partially open
14	very high		zero	open

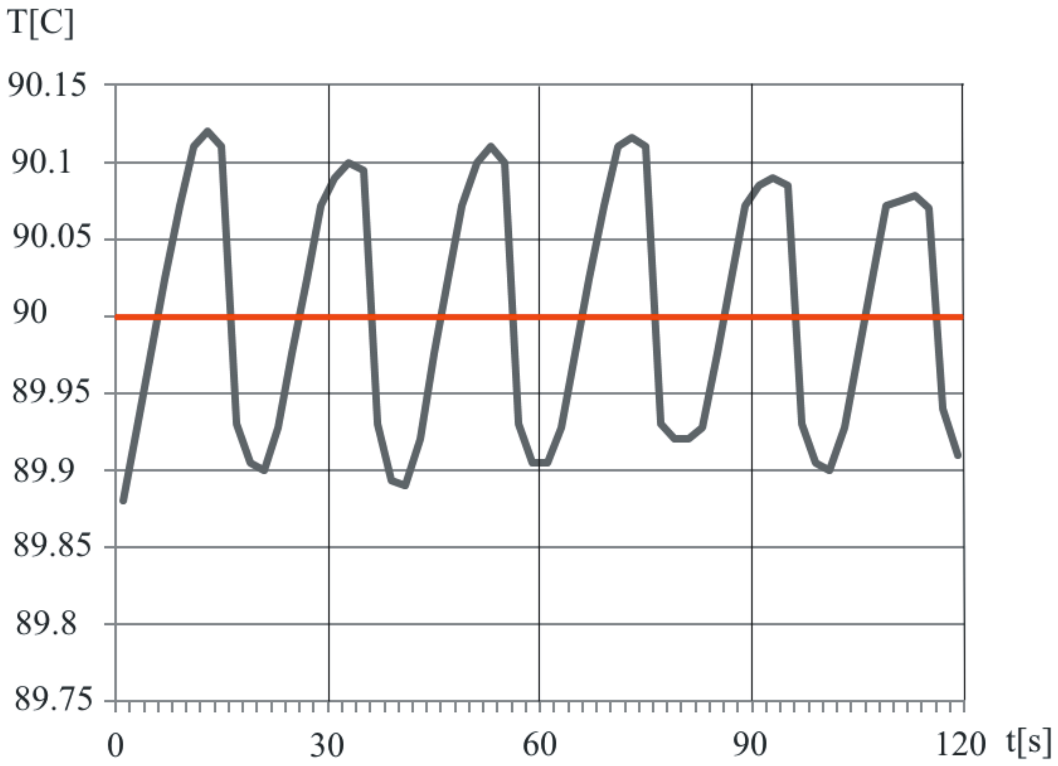
Control rules





Results


T[°C]
90
80
70
60
50
40
30
20
0
250
500
750
1000
1250
1500
t[s]


T[C]
90.15
90.1
90.05
90
89.95
89.9
89.85
89.8
89.75
0
30
60
90
120
t[s]

Development system and final product